

NHI Course No. 13063

# ***Seismic Bridge Design Applications***

25 April 1996

Part One

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16. Abstract Seismic Bridge Design Applications, Parts One and Two, contains the material used in two one-day national satellite seminars broadcast from the University of Maryland to provide seismic design application instruction. Mr. Robert Mast and Dr. Lee Marsh of BERGER/ABAM Engineers, Inc., were the instructors and developed the course materials. Part One includes seven sessions covering basic seismic principles, one complete seismic analysis and design example, modeling guidelines, multimodal analysis, and column design features. Part Two includes "homework problems" assigned after the first seminar as well as specific topics requested by participants of the first seminar.					
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# Seismic Design of Bridges, Seminar No.1 – Outline

Type of Material	Session No.	Topic
Background	1	Seismic Design Philosophy Seismic Hazard Analysis
	2	Structural Dynamics Response Spectra Overview of Division I-A
Worked Example	3	Two-Span Example Analysis
	4	Two-Span Example Design
Detailed Topics	5	Modeling Guidelines Foundation Modeling Multimode Analysis
	6	Multimode Analysis
	7	Intended Inelastic Behavior SPC B vs. SPC C and D Wall Pier Design Detailing Issues Questions and Answers

# **Session 1**

## **Seismic Design Philosophy**

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- **Code in 1973**
- **Lessons from Earthquakes**
- **Overall Objectives**



# AASHTO Through 1973

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## 1.2.20 — Earthquake Stresses

In regions where earthquakes may be anticipated,

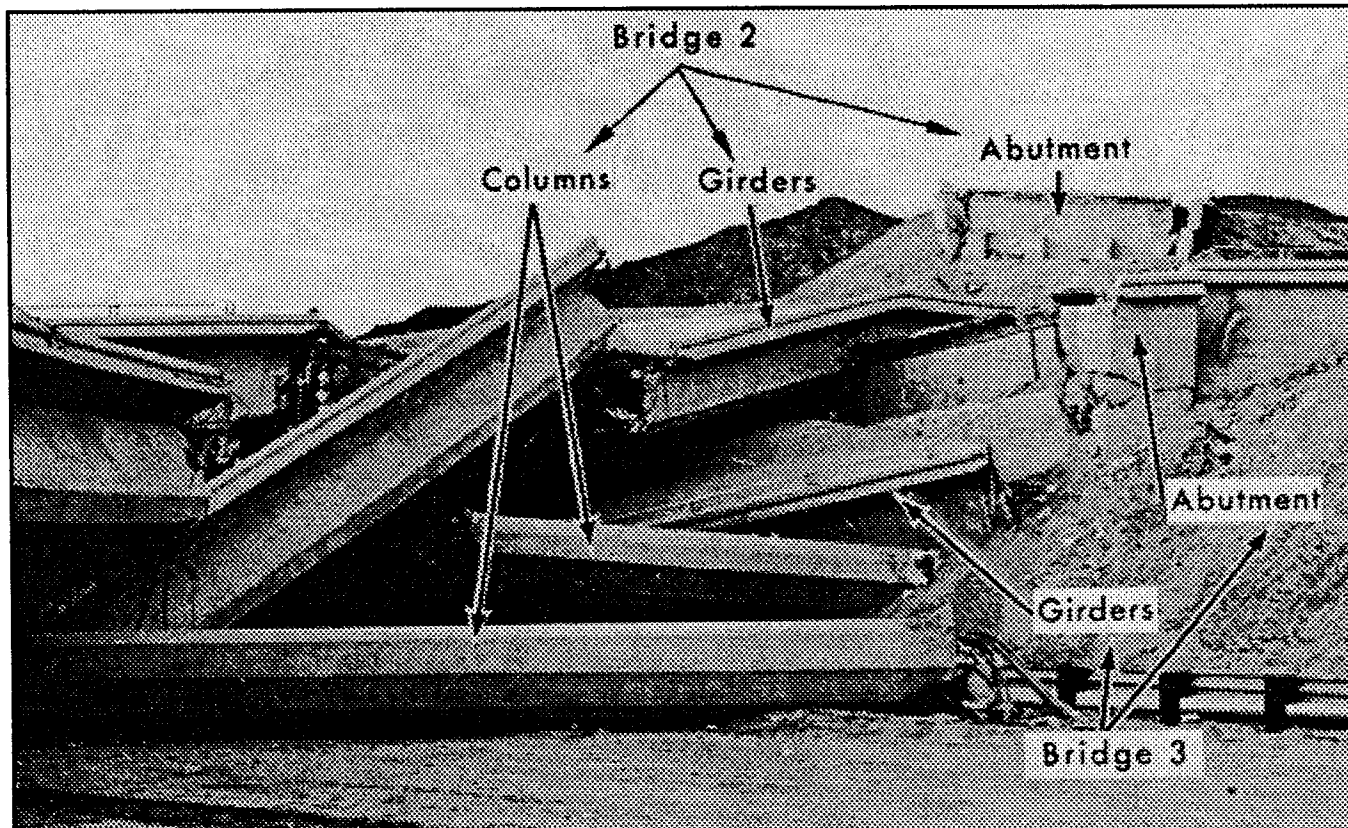
$$EQ = CD$$

EQ = Lateral Force Applied Horizontally

D = Dead Load of Structure

C = 0.02 for Structures on Material Rated as 4 Tons or  
More per Square Foot  
= 0.04 for Structures on Material Rated as Less than 4 Tons  
per Square Foot  
= 0.06 for Structures on Piles

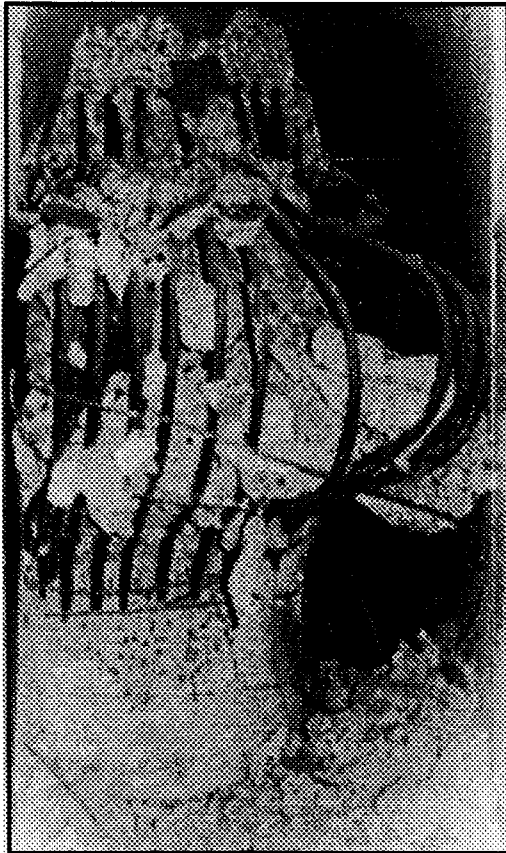
# 1971 San Fernando, CA



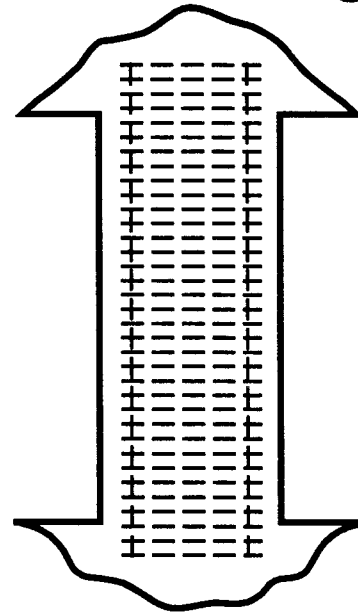
NBS

## Failure Type: Shear

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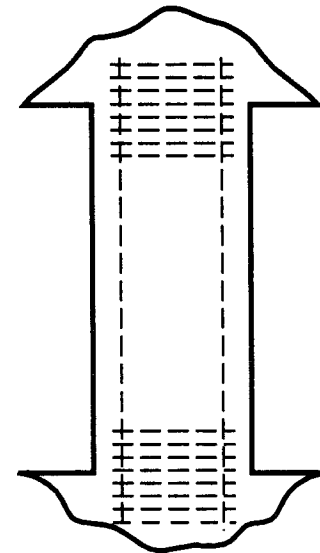
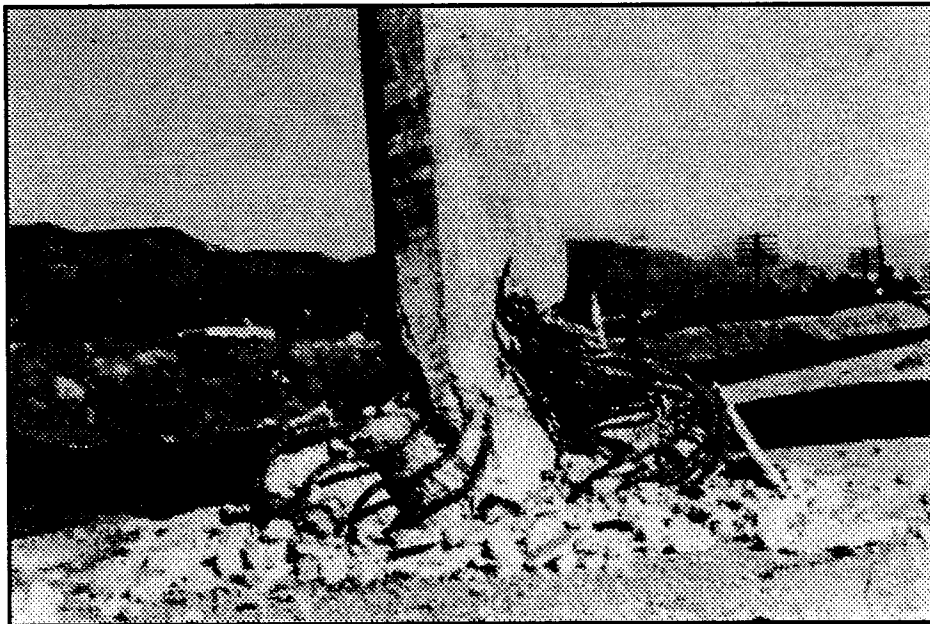
**Nature:** Brittle  
**Prevention:** Sufficient Shear Strength



# Failure: Bursting of Confinement

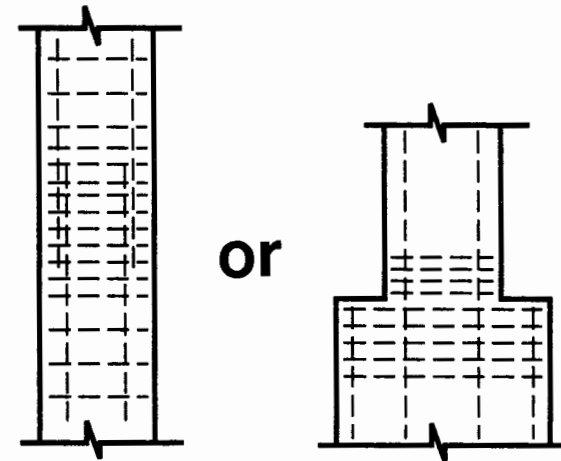
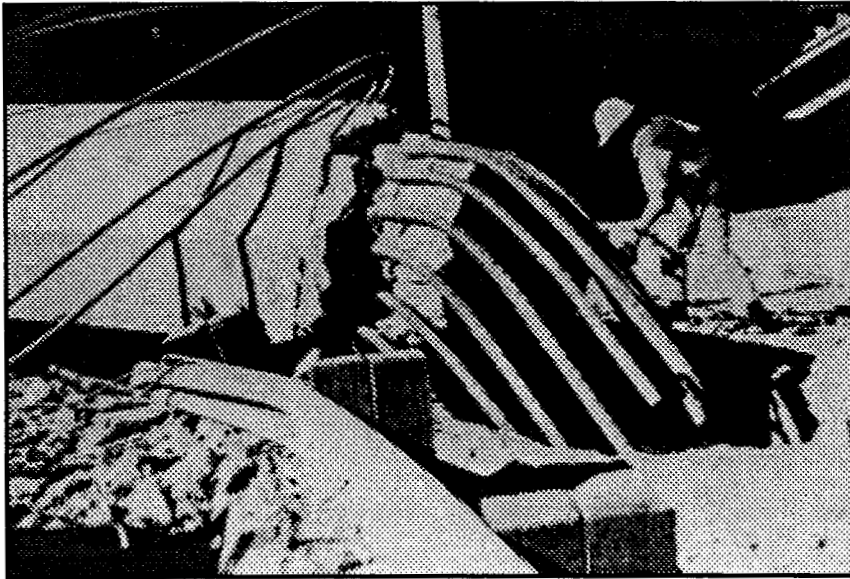
(Some Hinging then Shear Failure)

**Nature:** Limited Ductility  
**Prevention:** Adequate Hinge  
Zone Confinement



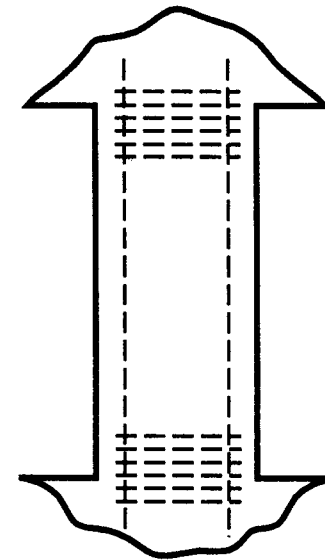
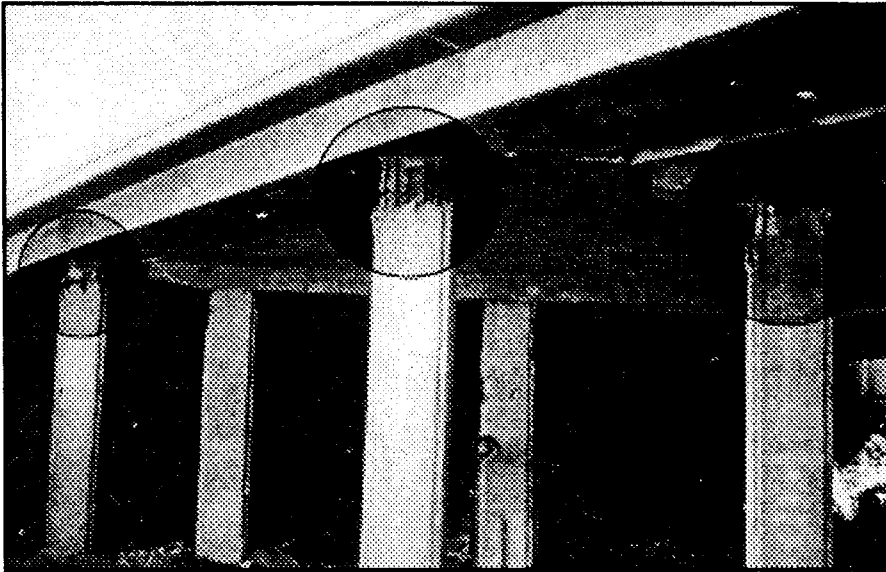
# Failure: Insufficient Development

**Nature:** Limited Ductility  
**Prevention:** Eliminate Splices  
in High Moment Zones  
or Confine Splice Heavily



# Behavior: Limited Flexural Damage

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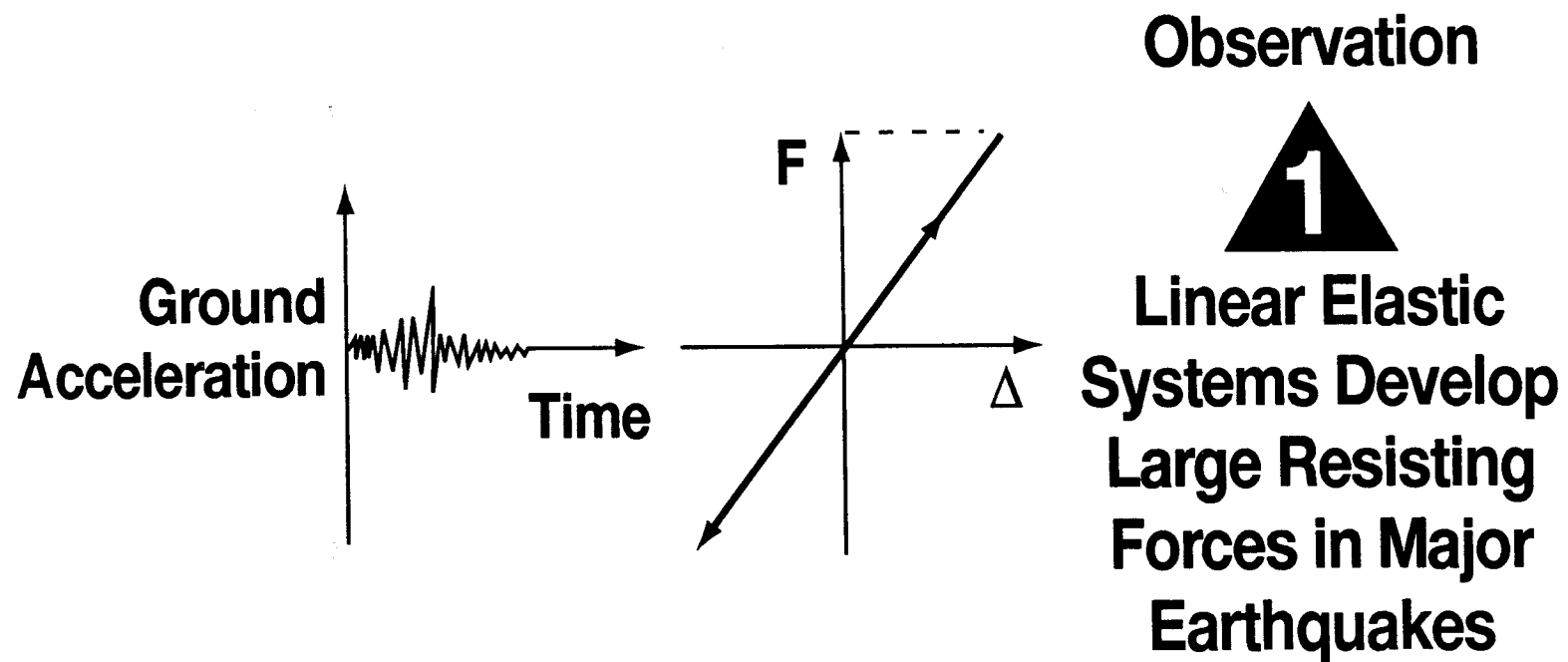


**Nature:** Ductile

**How to Obtain:** Sufficient Confinement to  
Prevent Crushing and Bar Buckling  
Also Suppress Shear, Pullout, and Stability Failure

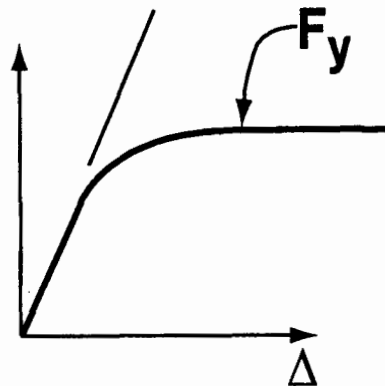
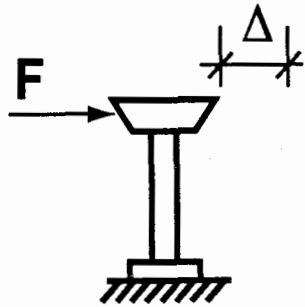
# Three Basic Observations

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## Three Basic Observations (continued)

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### Observation

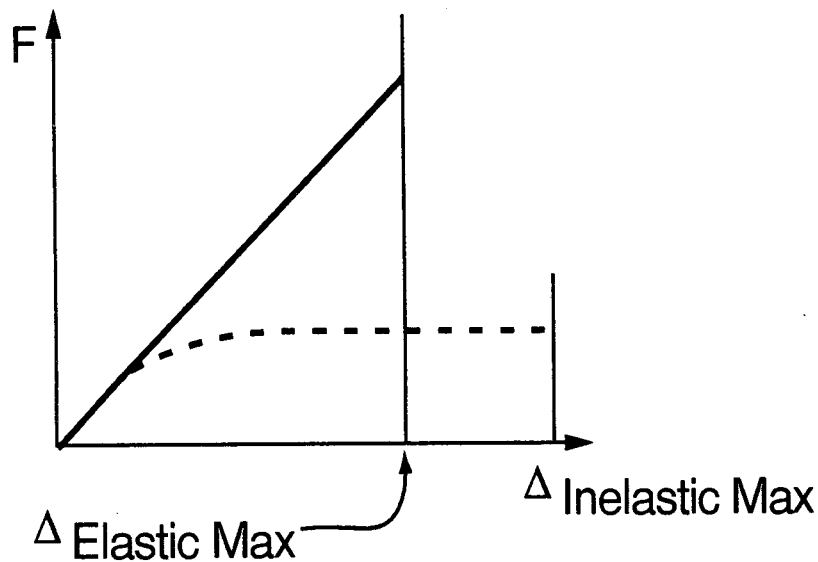
**2**

**We Can Build Ductile  
Structures (Ability to  
Deform into Inelastic Range)**



## Three Basic Observations (continued)

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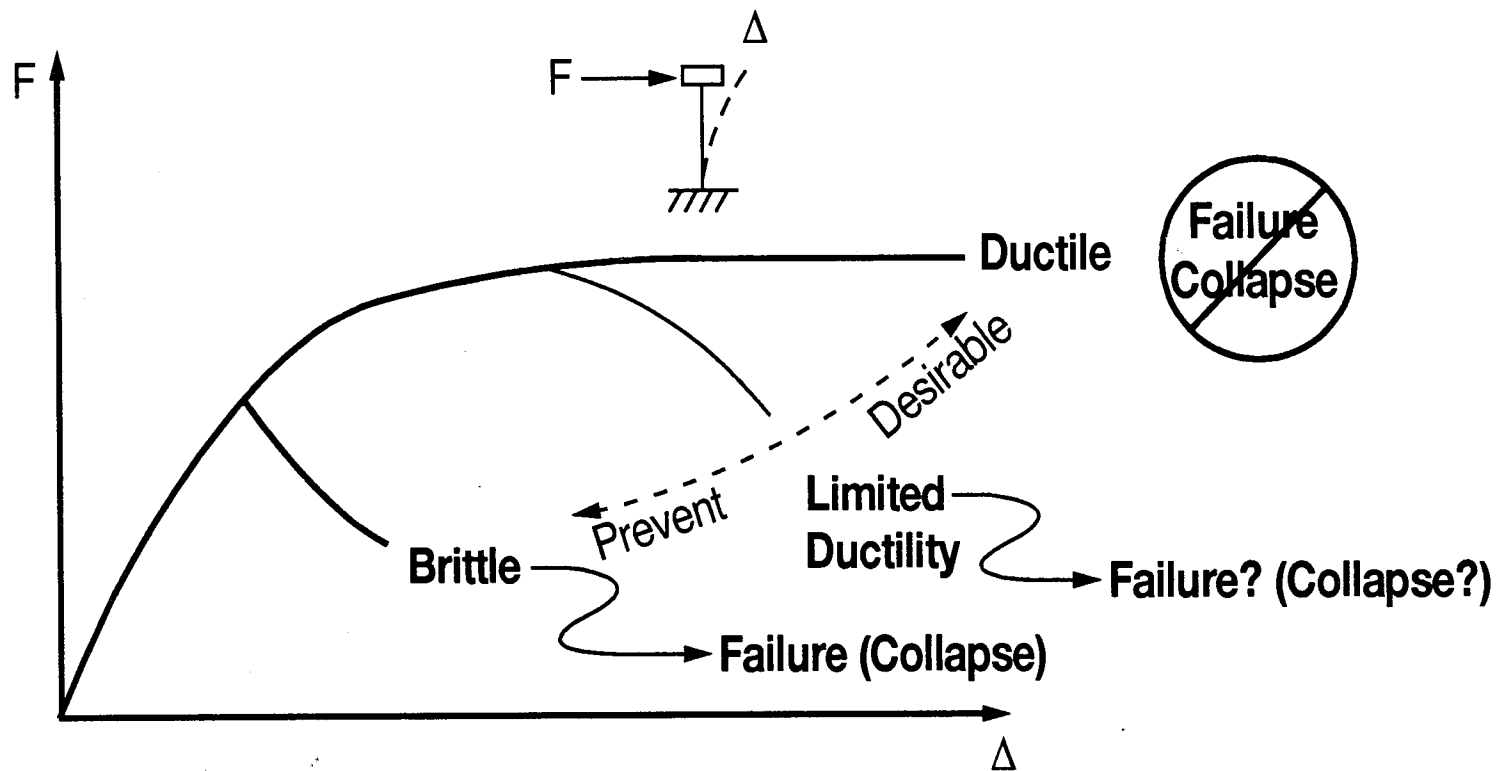


### Observation

**3**

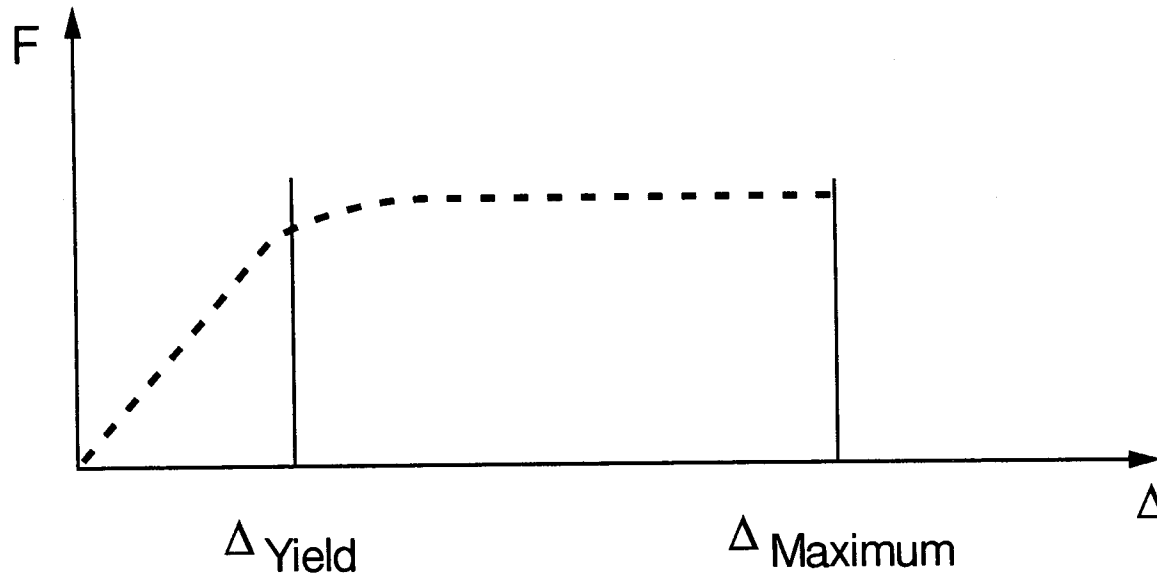
**Maximum Displacements  
of Elastic Systems and  
Similar Period Yielding  
Systems Are Roughly  
Equal**

# Types of Inelastic Behavior



# Ductility

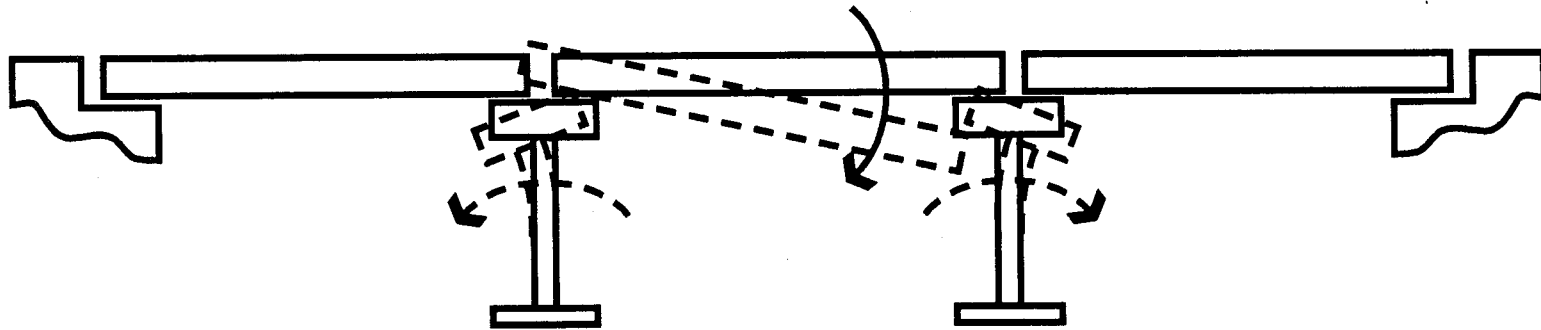
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$$\text{Ductility } (\mu) = \frac{\Delta_{\text{Maximum}}}{\Delta_{\text{Yield}}}$$

## Handling Displacements/Use Conservative Estimates

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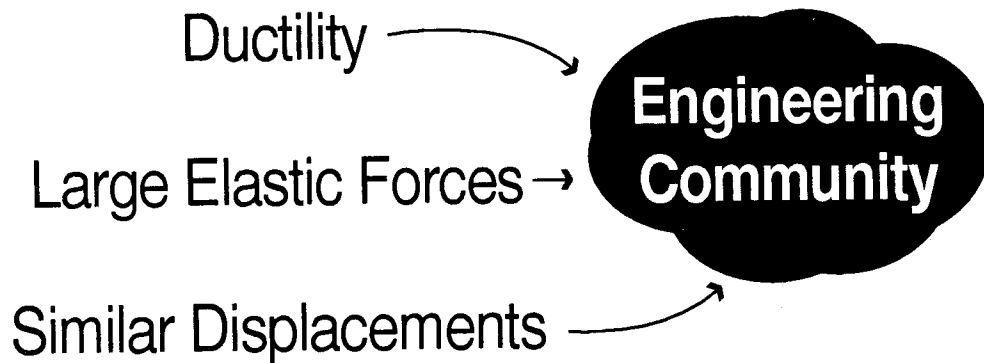
Seat Width Must include Allowance for

- Yielding
- Out-of-Phase Movement of Separate Units

# Seismic Design Philosophy

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## Observations



## Design Philosophy

- Allow Yielding (Damage) in Major Earthquake
- Damage Should Be Accessible
- No Collapse

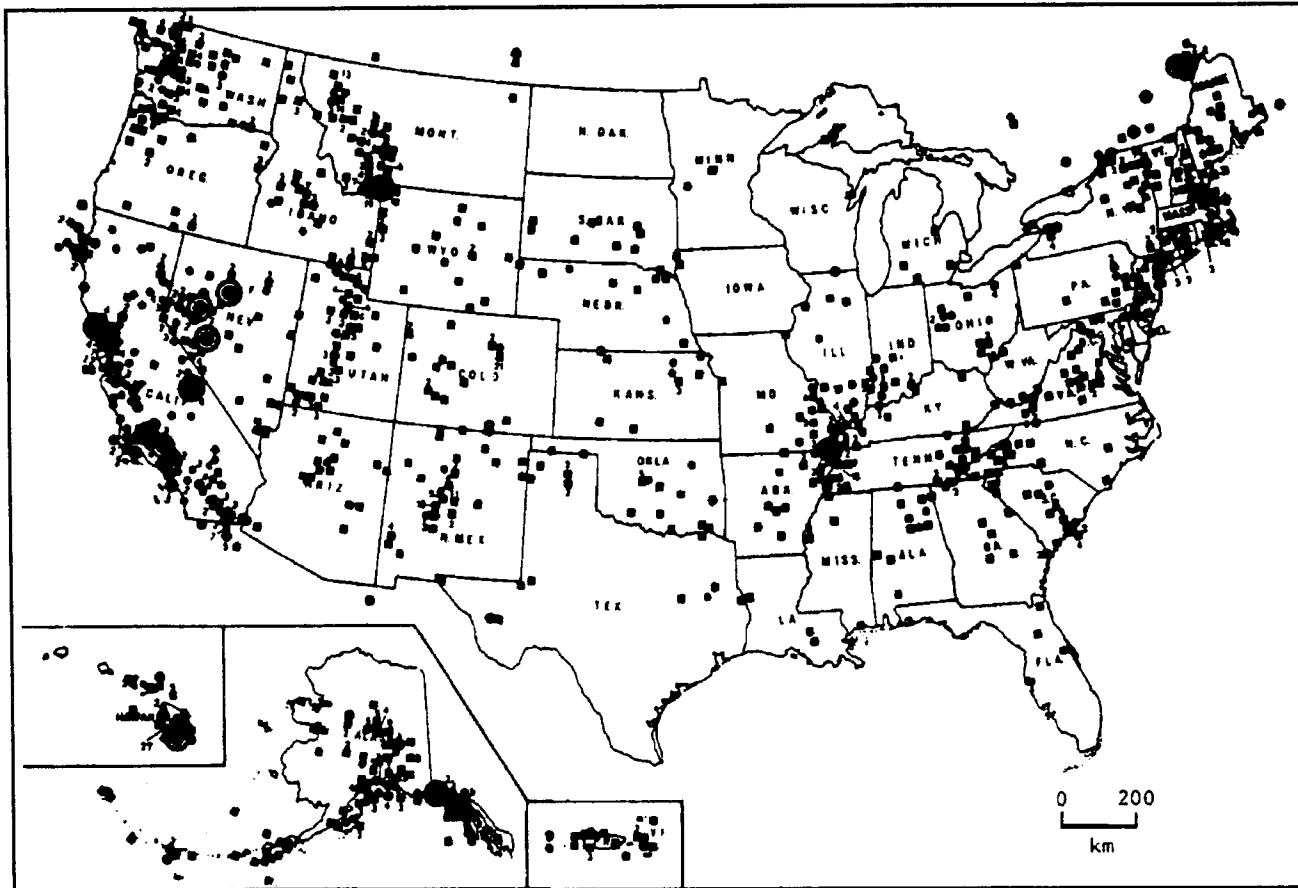
# **Session 1**

## **Seismic Hazard Analysis Concepts**

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- **Regional Importance**
- **How the Ground Moves**
- **Where the Seismic Hazard Maps Come From**

# Earthquake Occurrence in United States



Algermissen

# Charleston, South Carolina / 1886

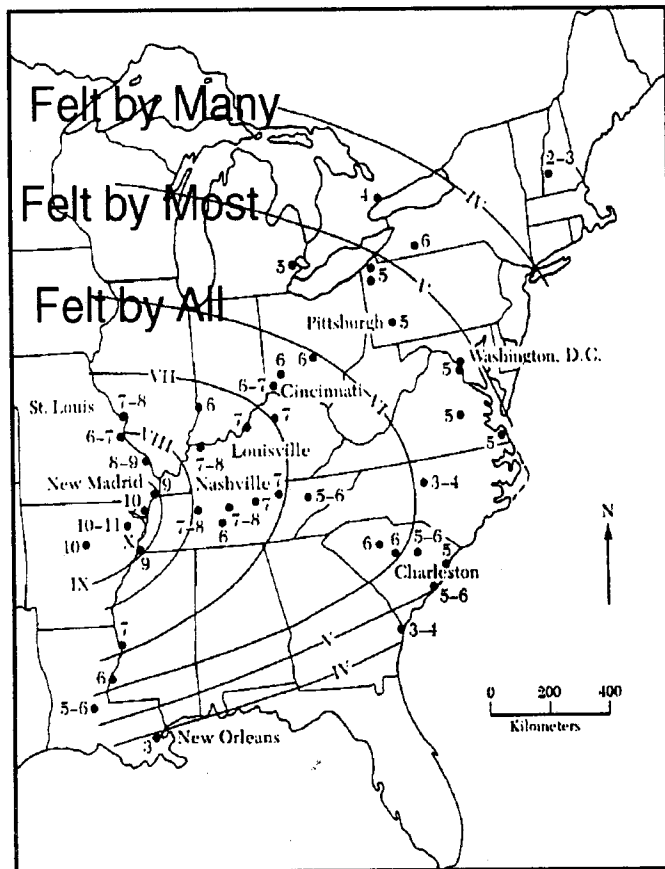
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Magnitude = ?? Felt in Boston, Chicago, and  
St. Louis (All ~ 900 Miles Away)





# New Madrid, Missouri / 1811 – 1812



Bolt

## 3 Main Earthquakes

- Magnitudes ~ 7.3 to 7.8

December 1811

January 1812

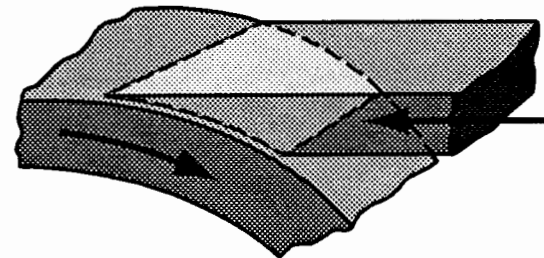
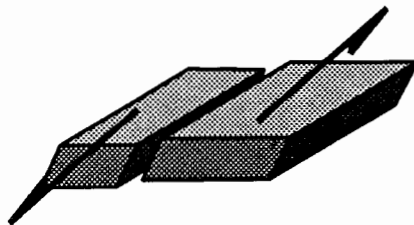
February 1812

- Chimneys Down in Cincinnati, Ohio
- Falls Formed in Mississippi River

# Earthquake Occurrence and Sources

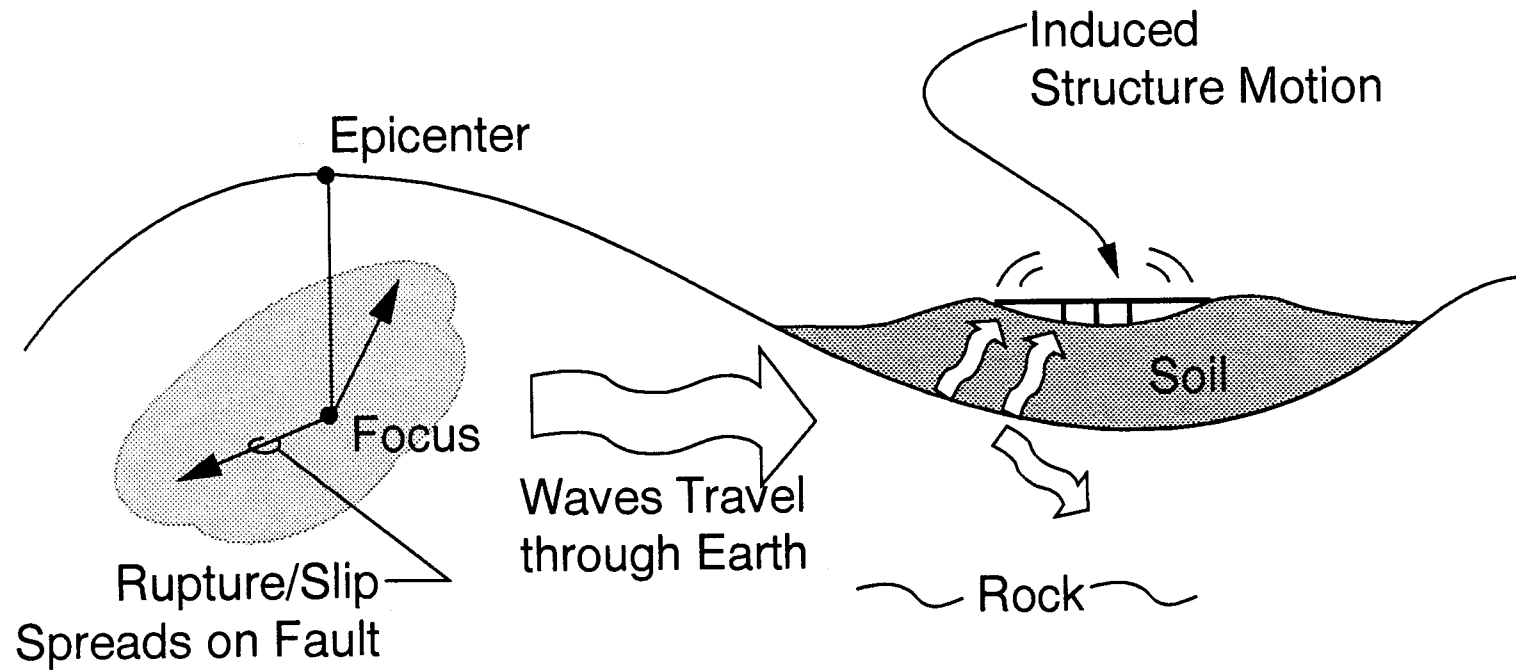
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- **Earthquake Occurrence:**  
Primarily at Plate Boundaries — California  
Some Occur within Plates — South Carolina,  
Missouri, etc. — But Not as Often
- **Sources Can Be Identified as:**
  - Line (Faults)
  - Area (Subduction Zones)



# Earthquake Shaking / Sources-to-Site

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**Magnitude and Duration Proportional  
to Area and Amount of Slip**

# Characterizing Ground Motion for Design

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- Use Ground Acceleration
- Need to Go From Earthquake Source to Site Ground Acceleration
- Account for Known Rate of Occurrence

## **AASHTO I-A:**

'Probabilistic'  
Ground Motion

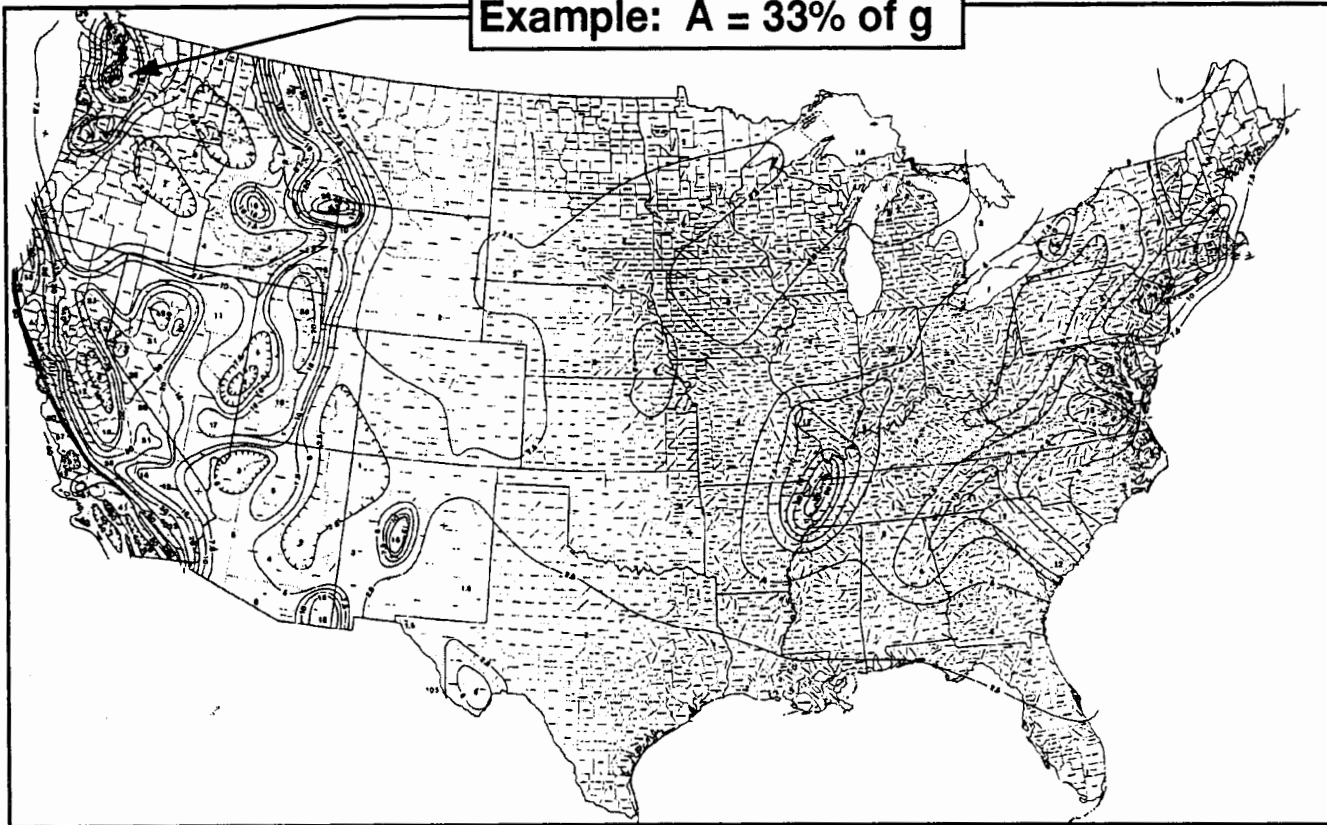
## **Example:**

There Is a 10% Chance that  
an Earthquake Will Produce  
an Acceleration that Exceeds  
0.33g at a Site During Any  
50-Year Interval

# AASHTO 1-A / Acceleration Coefficient, A

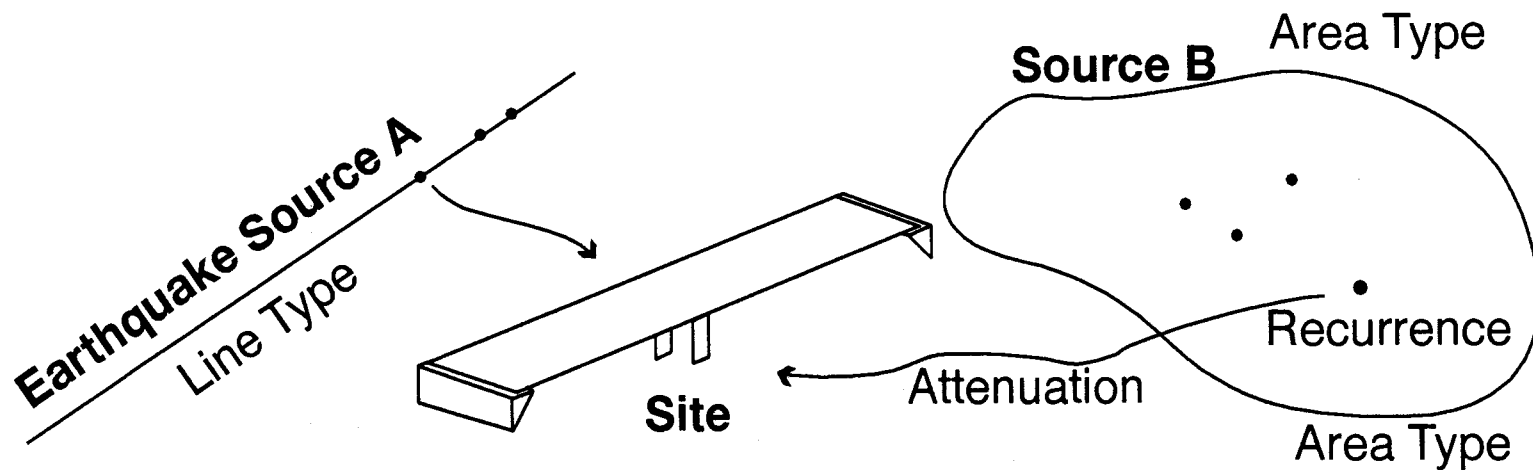
A Is Given as Percentage of Gravity

Example:  $A = 33\%$  of  $g$



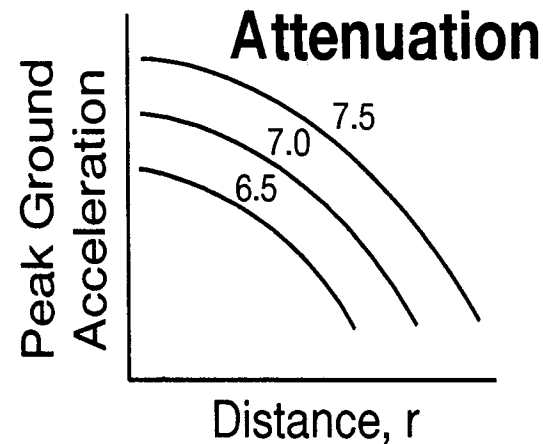
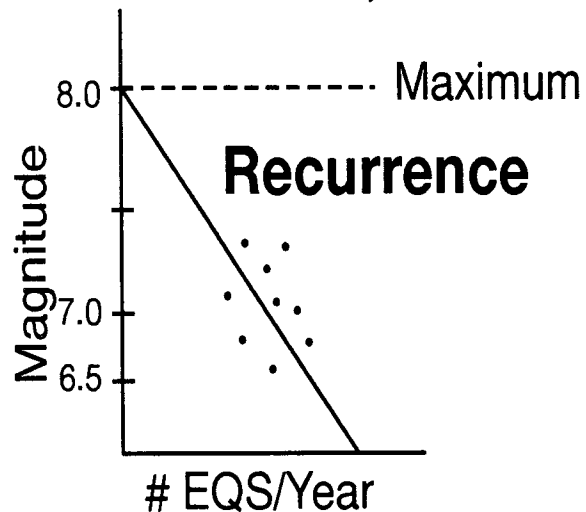
# Probabilistic Ground Motion

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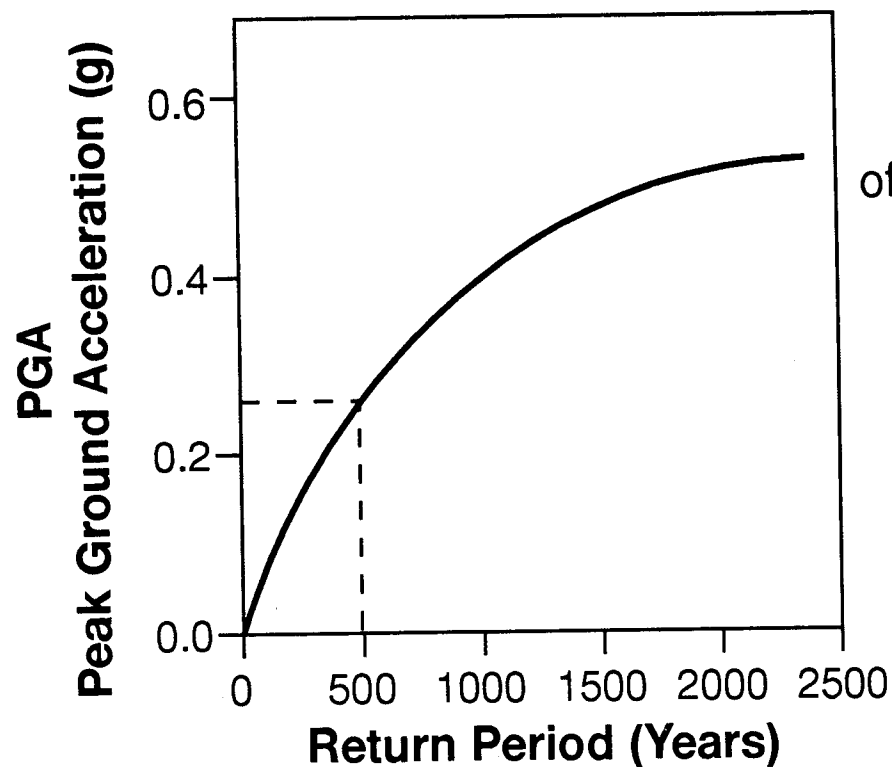
# Probabilistic Ground Motion (continued)

- For Each Source, Know



- For All Sources and All Locations within Source,  
Add Up Probability that an Earthquake Produces an Acceleration  
Greater than a Specified Value at the Site for a Given Time Interval

# Product of Hazard Analysis



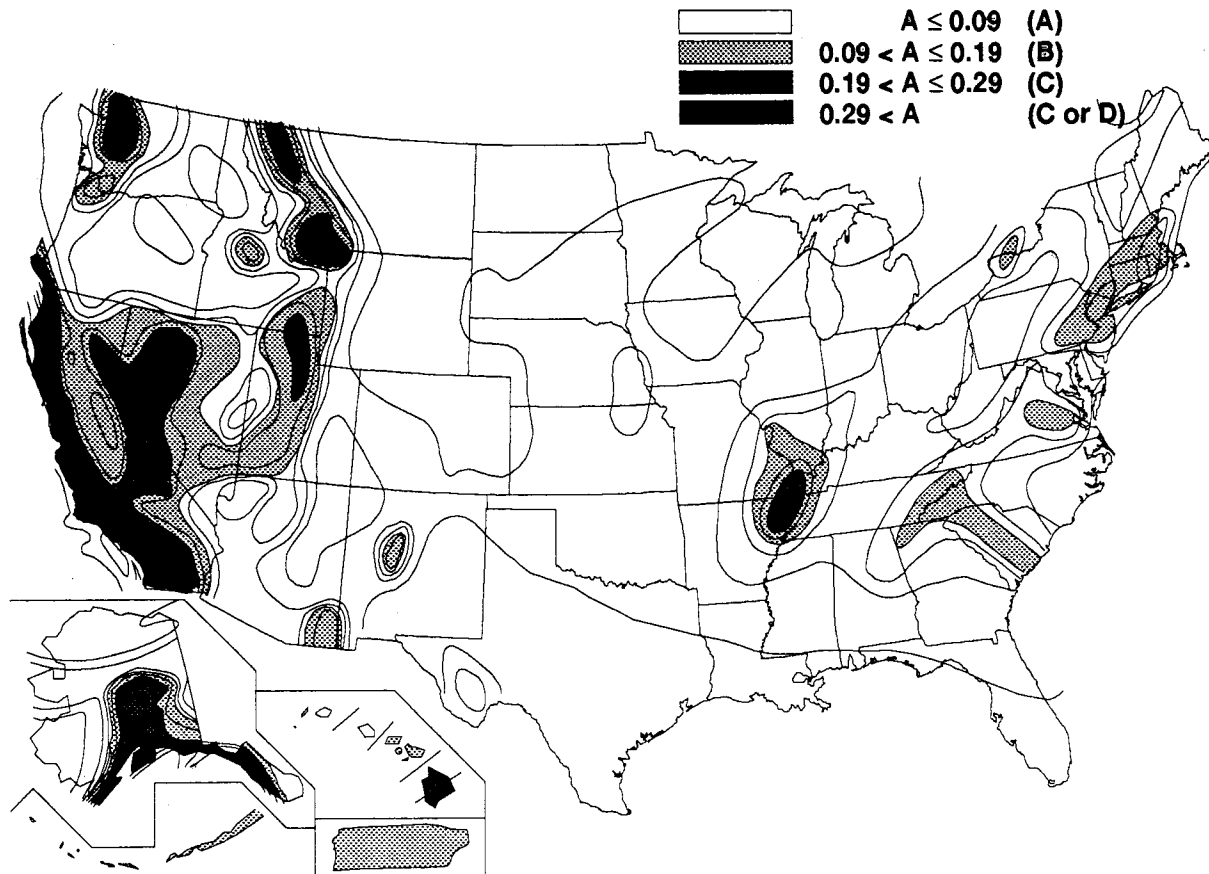
$$\text{Probability of Exceedence} \approx \frac{\text{Time}}{\text{Return Period}}$$

$$0.10 = \frac{50 \text{ Years}}{500 \text{ Years}}$$

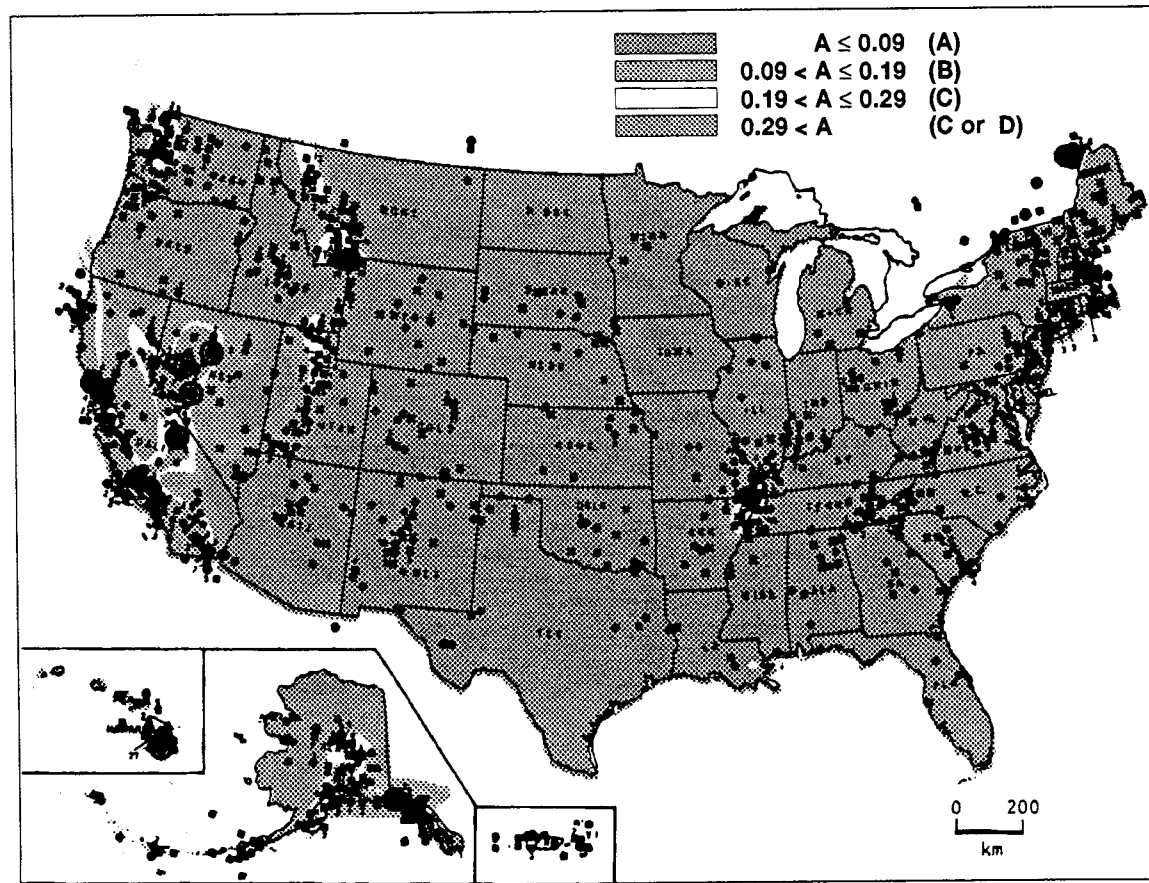
(Applies for  
Low Probabilities)



# AASHTO 1-A Acceleration Map



# AASHTO Map vs. Occurrences



Adapted from Algermissen, 1983, and AASHTO, 1995

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UMD-ITV

Seismic Bridge Design Applications

25 April 1996, NHI Course Code No. 13063



## **Session 2**

# **Structural Dynamics Concepts**

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- **Single-Mass Systems**

Free Vibration

Damping

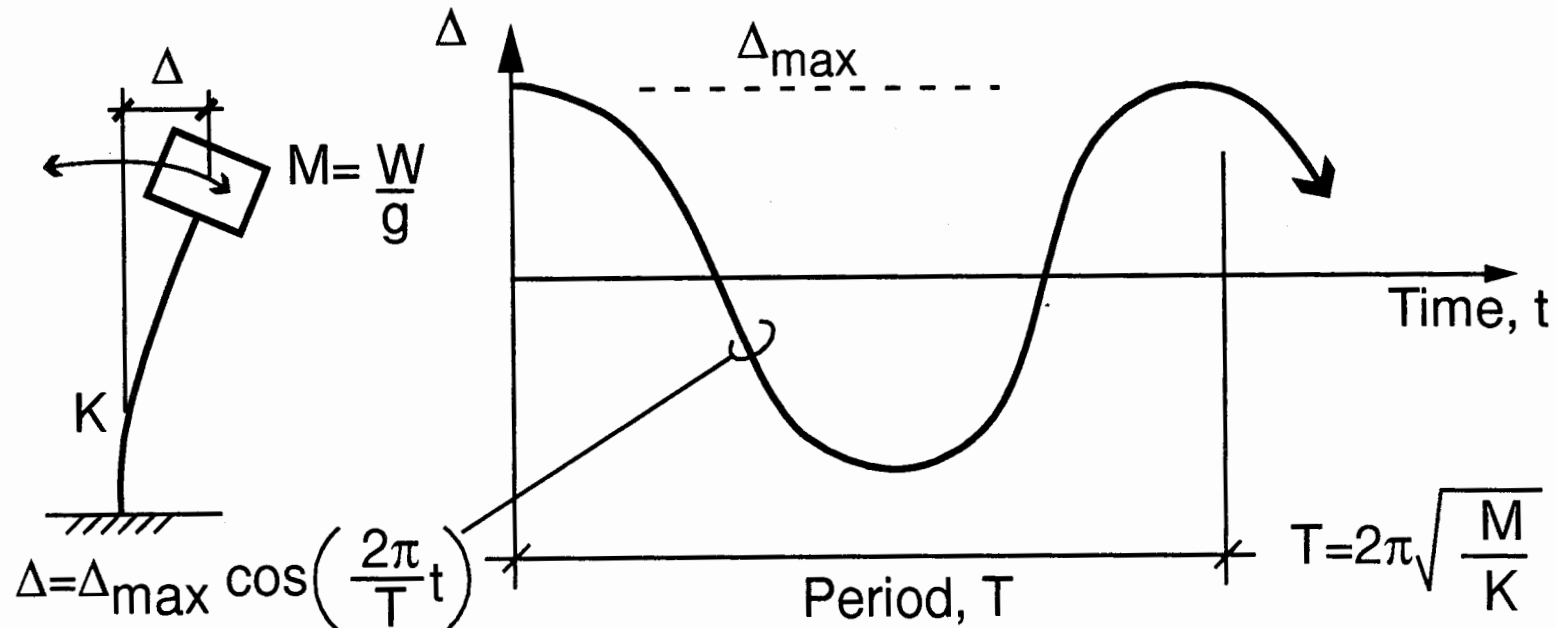
Forced Vibration

Earthquake Response

- **Multiple-Mass and Distributed-Mass Systems**
- **Response Characterization**

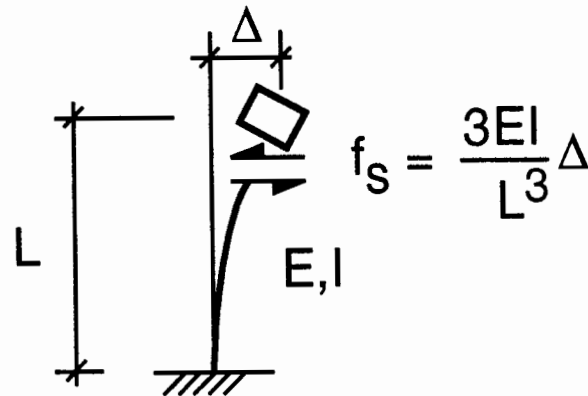
# Free Vibration

No Applied Force /  
Initial Displacement then Release

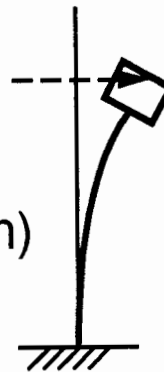


# Internal Forces / Free Vibration

Resisting Force  
(Spring Force)



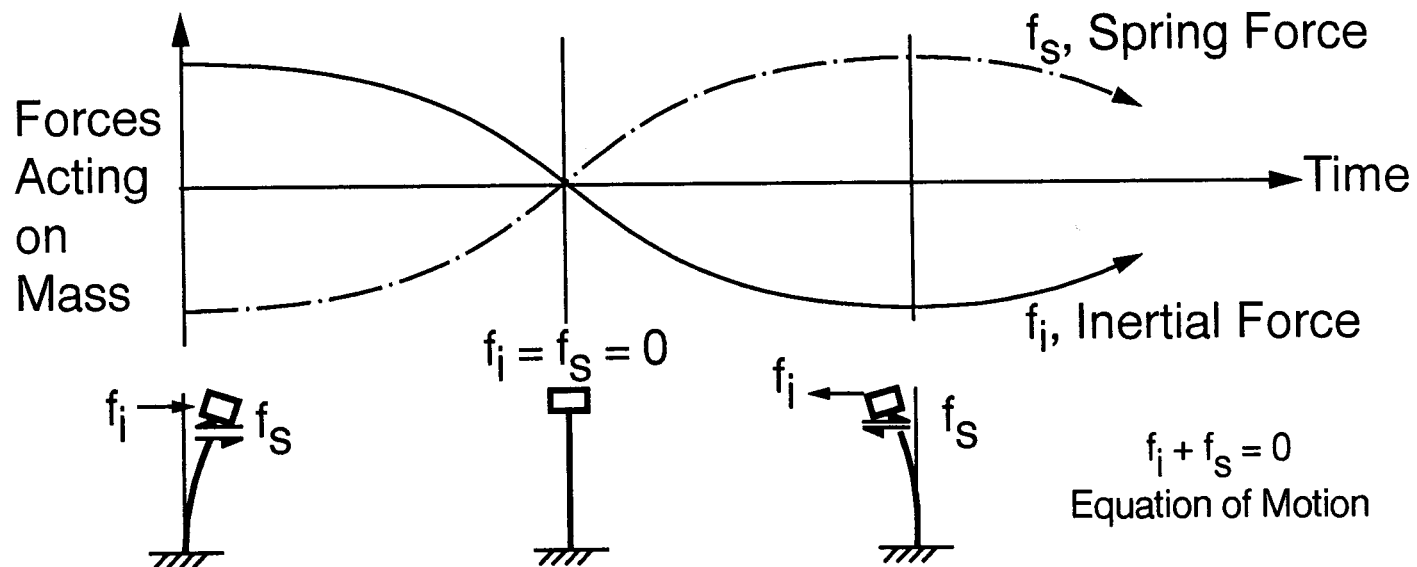
Inertial Force  
Newton's 2nd Law  
(Force = Mass • Acceleration)



$$f_i = ma = m \frac{d^2 \Delta}{dt^2}$$

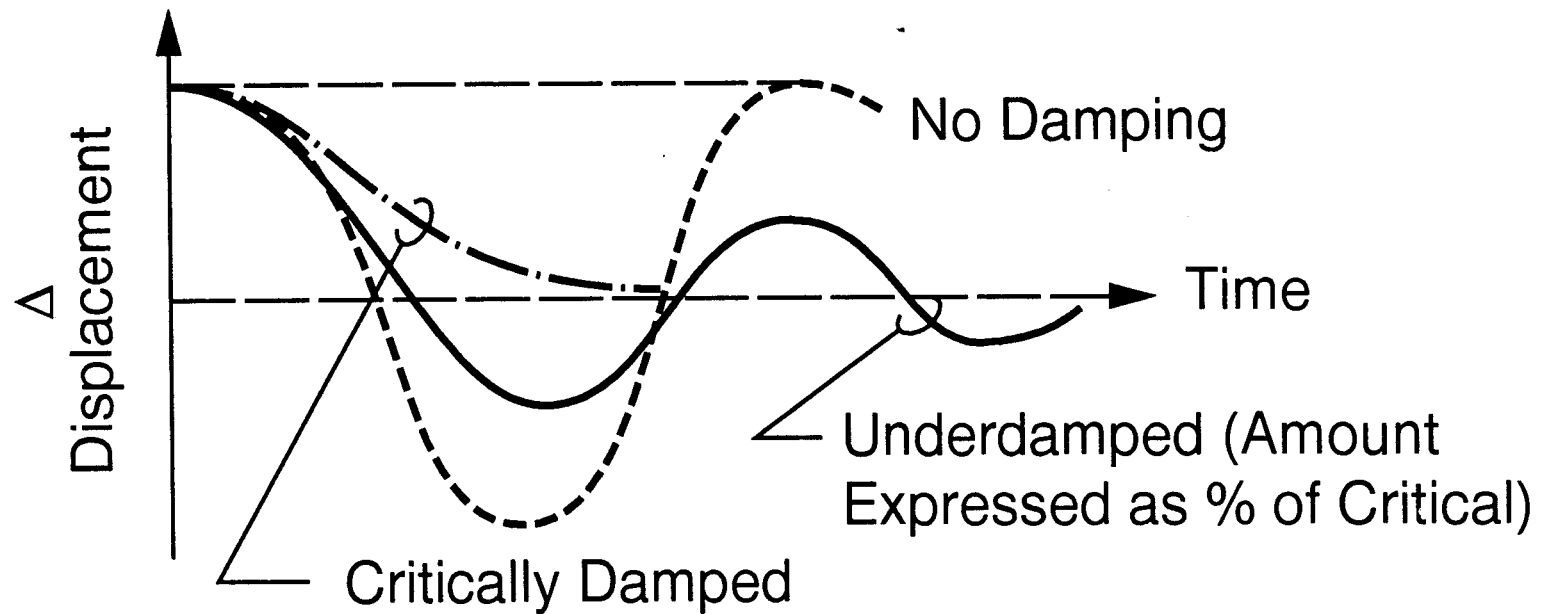
2nd Derivative  
of Displacement

# Dynamic Equilibrium / Free Vibration



- Structure Vibrates at Period,  $T$ ; Only 'Vibration Rate' for which Equilibrium Is Satisfied

# Damping

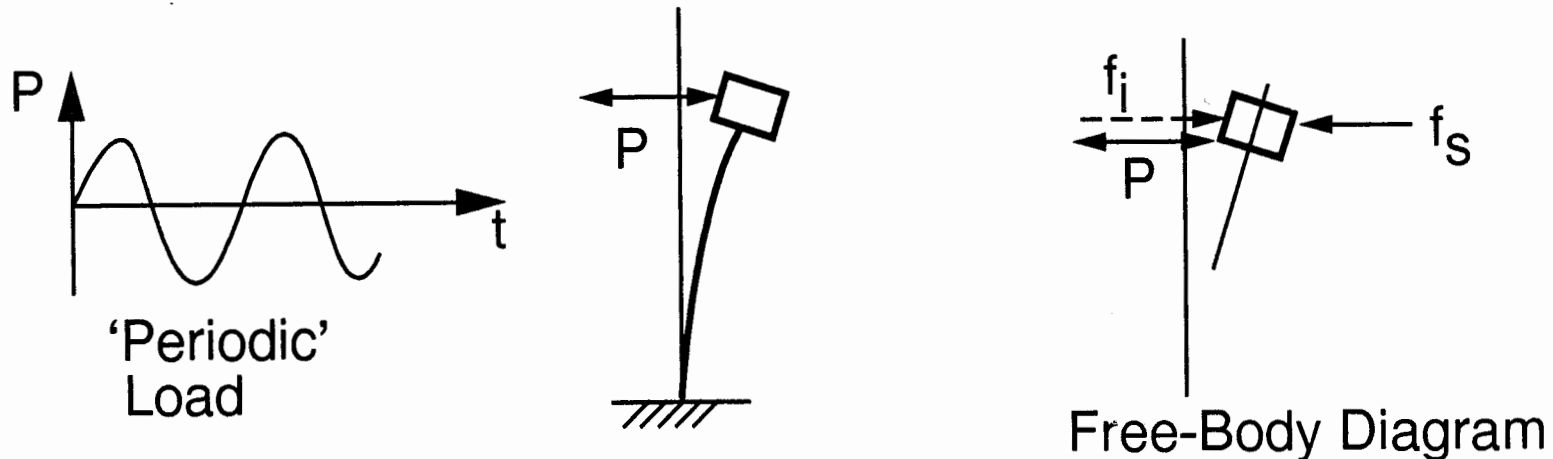


Typical Damping: Value  $\sim 5\%$



# Forced Vibration

Add a Time Varying Load to Our System / No Damping



Equation of Motion

$$P + f_i = f_s$$

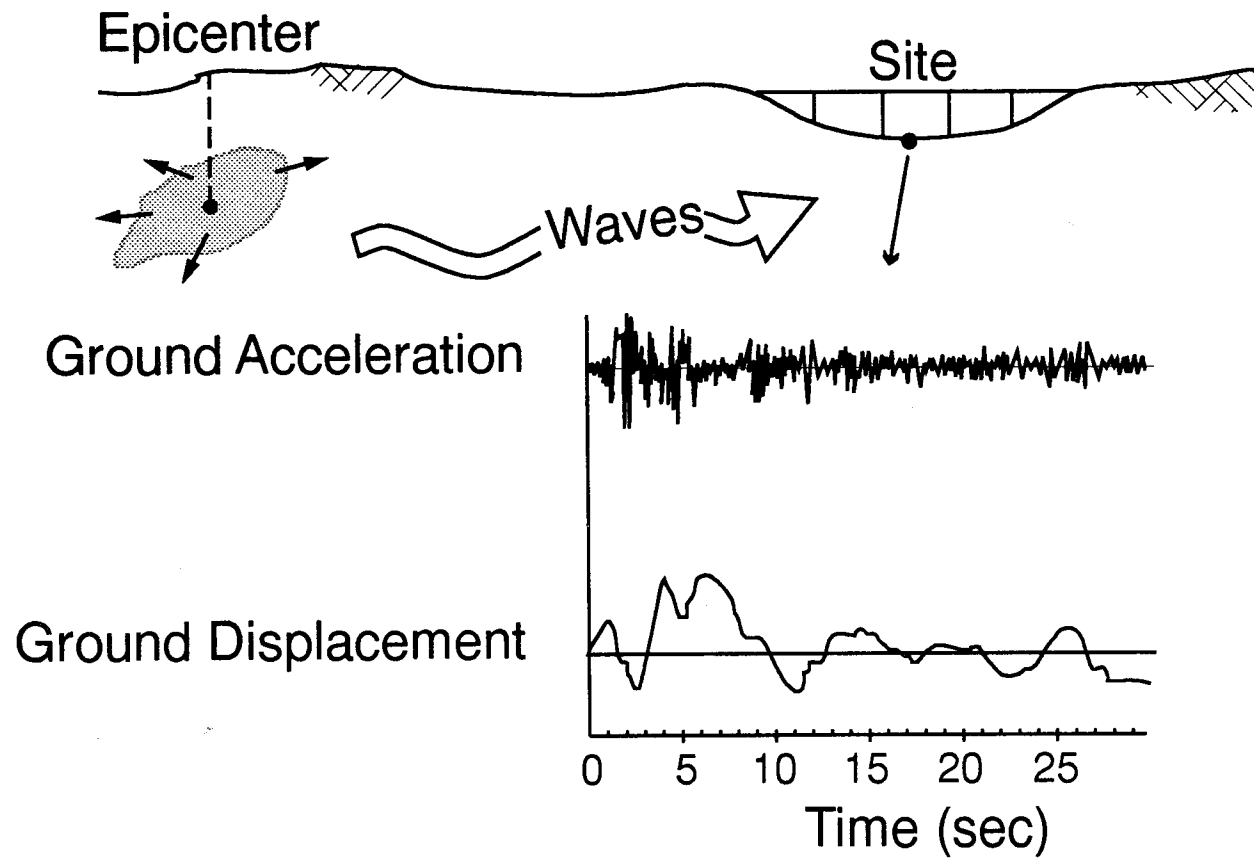
(Algebraically, the Signs Can Differ)

# Response Bounds / Periodic Loading

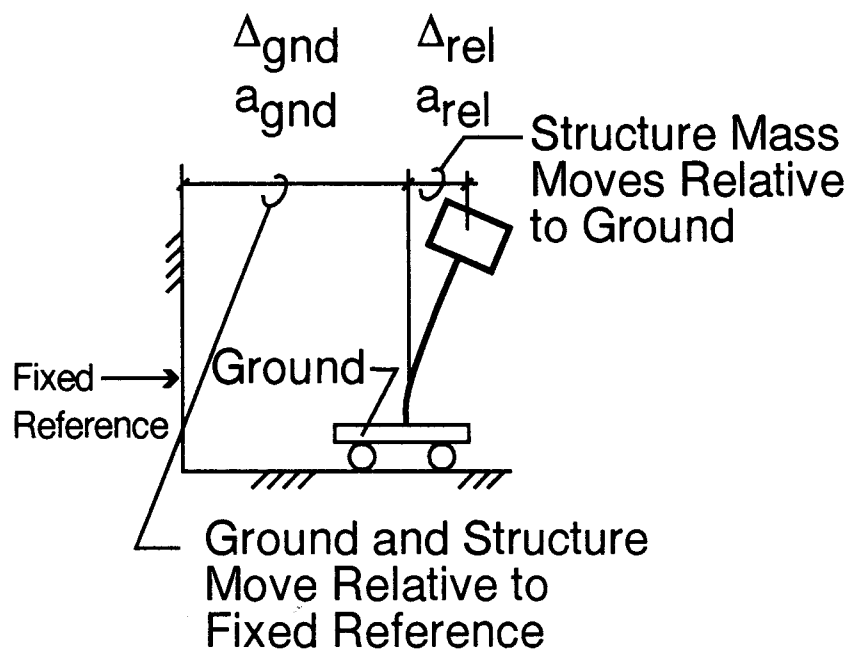
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- P Applied 'Slowly' – Response Is Essentially Static  
(Relative to Period)
- P Applied 'Rapidly' – Response,  $\Delta$ , Is Small
- Intermediate Case – Response Can Be Large  
'Amplification' → Resonance  
(As Loading Period Approaches  
Structure Period)

# Earthquake Effects



# Earthquake 'Loading' – Snapshot



## Inertial Force:

$$f_i = m(a_{gnd} + a_{rel})$$

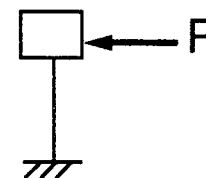
Total Acceleration  
of Mass

## Equilibrium:

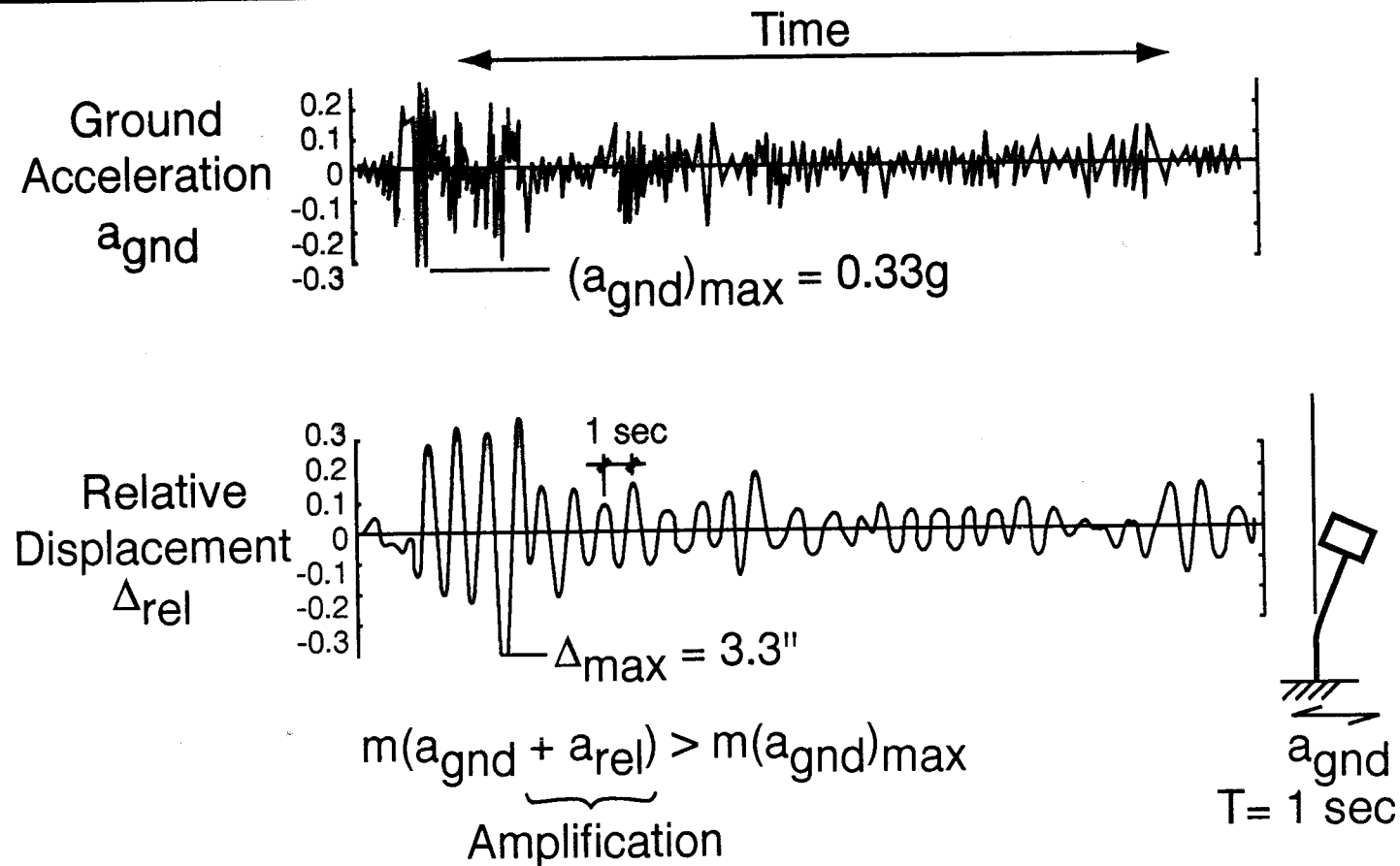
$$f_i = f_s$$

$$ma_{gnd} + ma_{rel} = k\Delta_{rel}$$

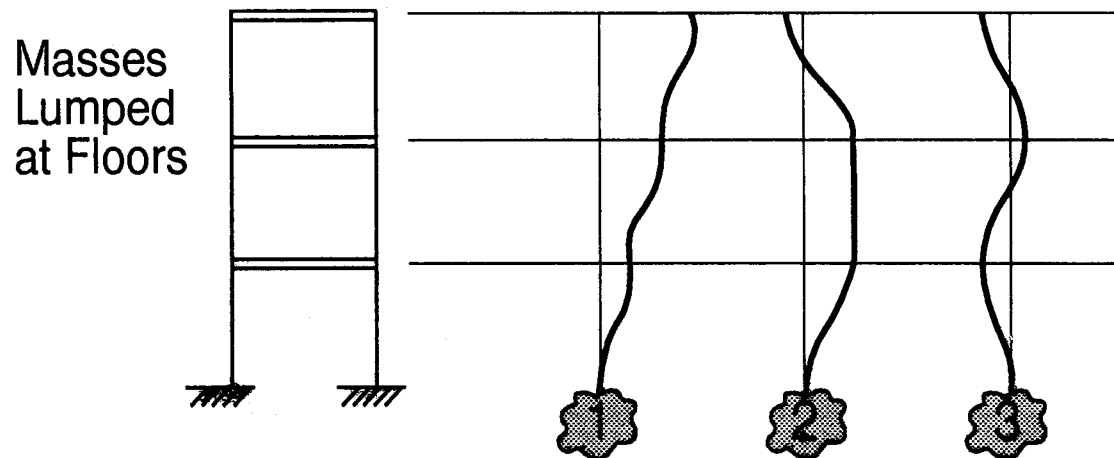
Equivalent to a 'Load',  $P$



# Earthquake Response




# Response with Distributed Mass Or Multiple Masses



- More than One Mass Leads to More Vibration 'Modes' and 'Periods'
- Modal Periods,  $T_i$  = Function of Shape  $i$  and  $T_1 > T_2 > T_3$  etc.

# Multi-Modal Response Basics

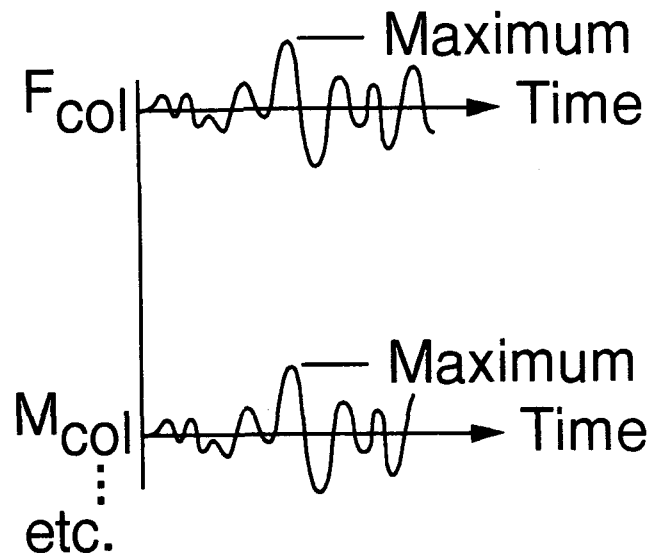
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- Number of Modes Set by Number of Masses and Their Freedom to Move (Dynamic Degrees-of-Freedom,  $n$ )
- Equilibrium Satisfies  $n$  Simultaneous Equations 
- Use Computer, but Understand Conceptually!
- Response (Linear Elastic) Is Superposition of  $n$  Modal Responses  
Forces,  $F$   
Displacements,  $\Delta$
- Not All Modes Are Required to **Estimate** Response

# How to Characterize Response

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- **Complex Method**  
Full Time History



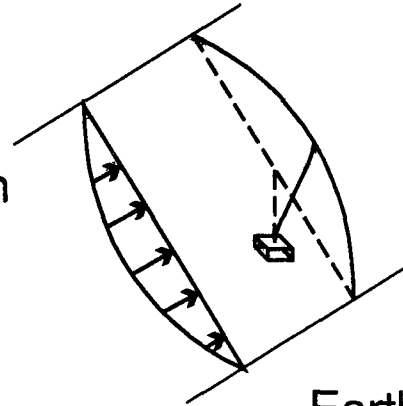


# How to Characterize Response (continued)

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- **Simple Method**

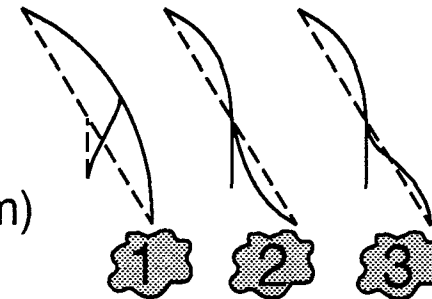
Quasi-Static (**Maximum** of One Approximate Mode Often Sufficient)



Earthquake Load  
(including Amplification)

- **Intermediate Method**

Multimode Superposition  
(Find Maximum of  $n$  Actual Modes,  
then **Combine** to **Estimate** Actual Maximum)



## **Session 2**

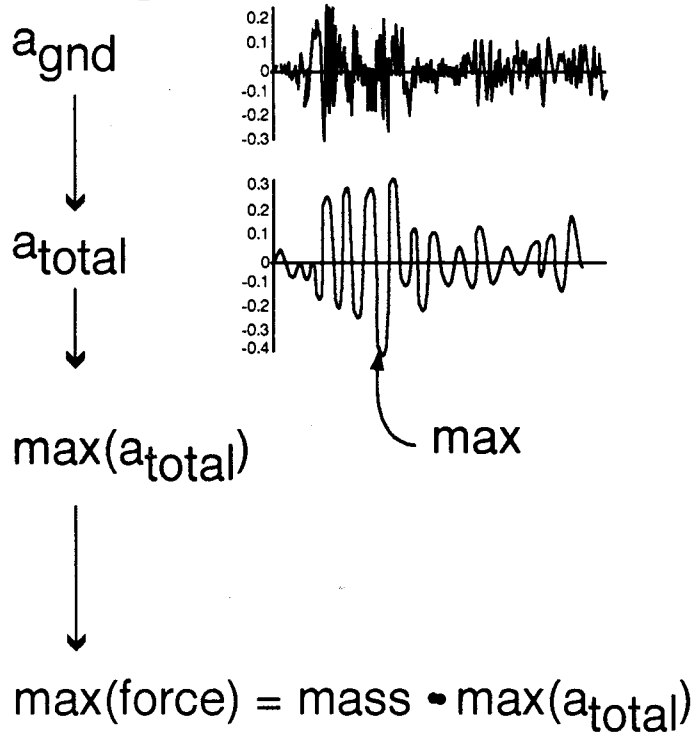
# **Response Spectra**

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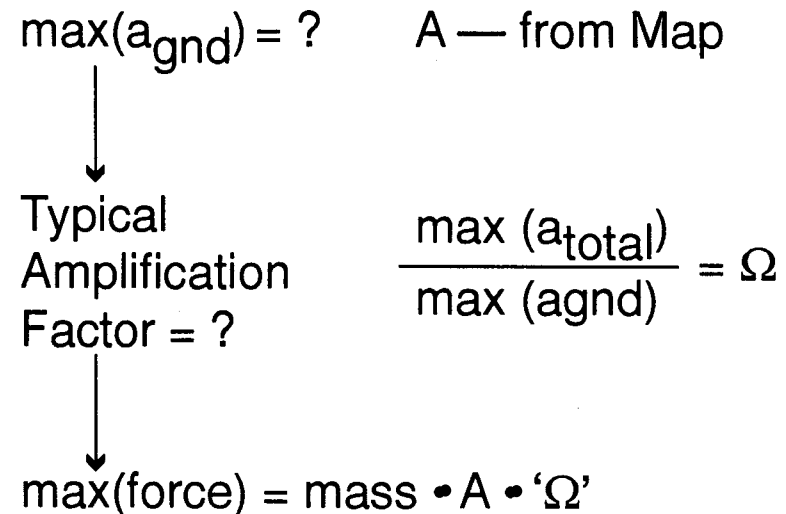
- **Definition**
- **Amplification and Period**
- **Effects of Site Soil Condition**
- **AASHTO Design Spectra**
- **How to Use Spectra**

# From Earthquake to Design

## Single Earthquake



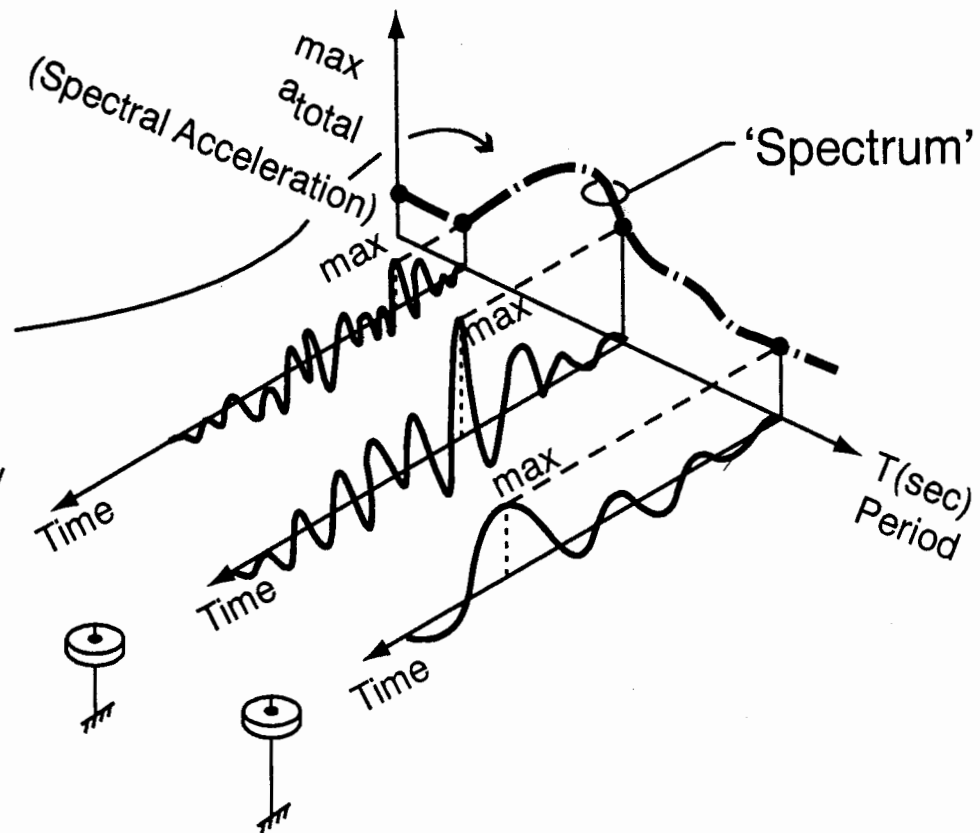
## For Design



# Define Response Spectrum

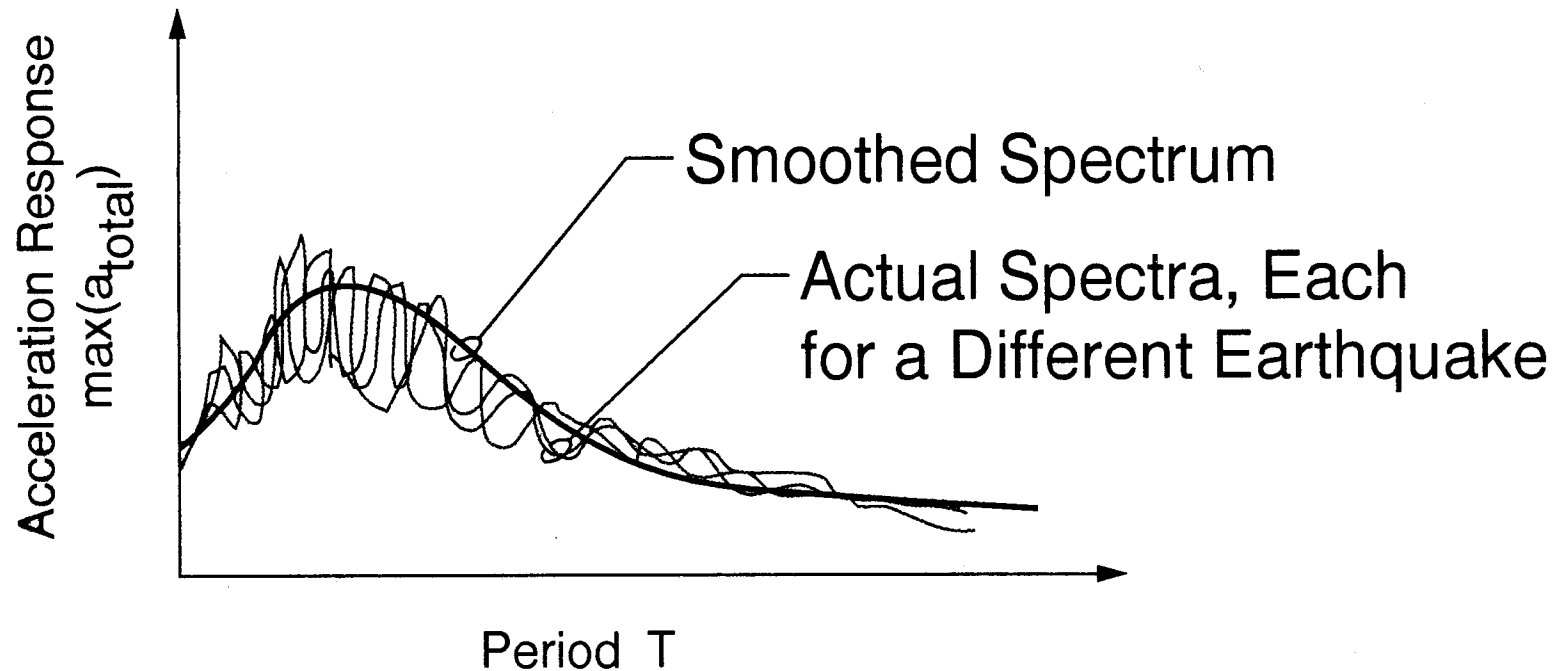
Determine Maximum Response for a Group of Structures, All with Different Periods; then Plot

All Structures Subject to the Same Ground Motion

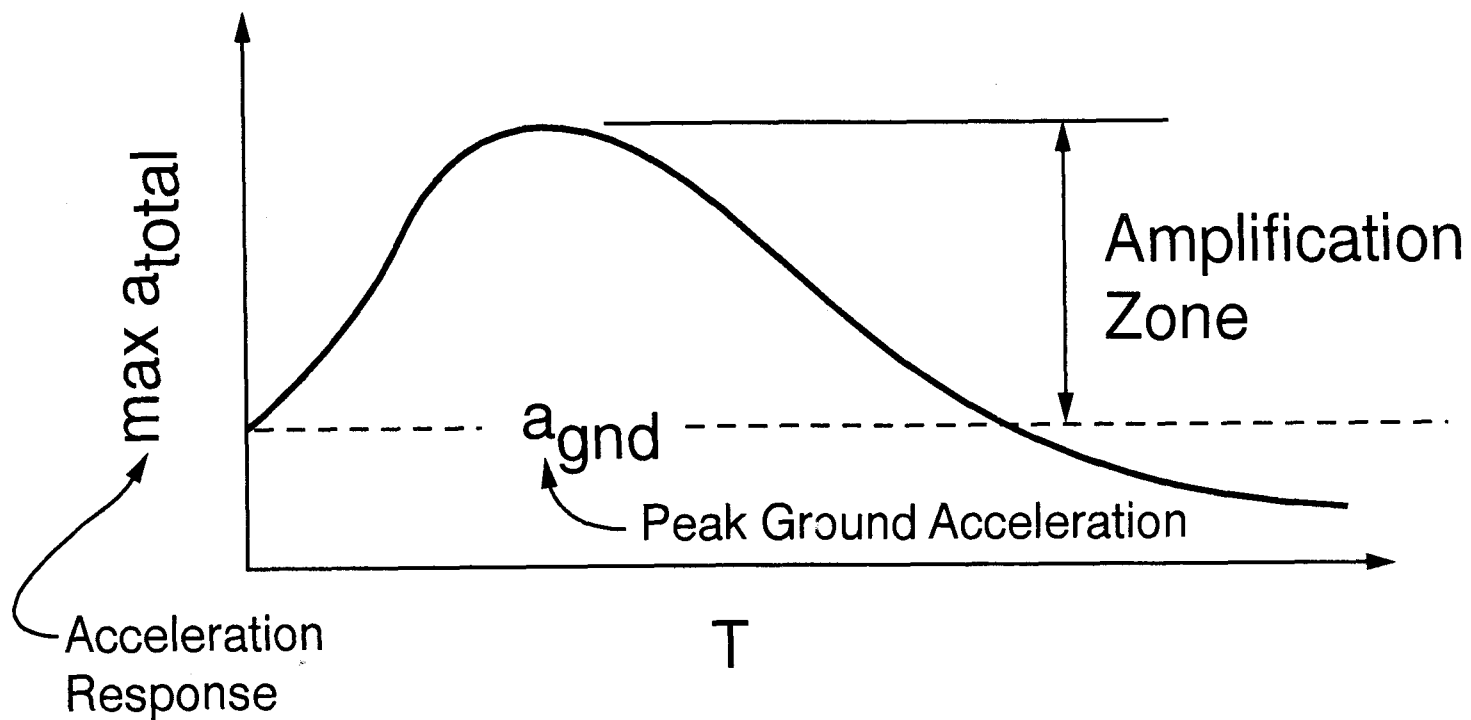


# Actual Spectra vs. Smoothed Spectrum

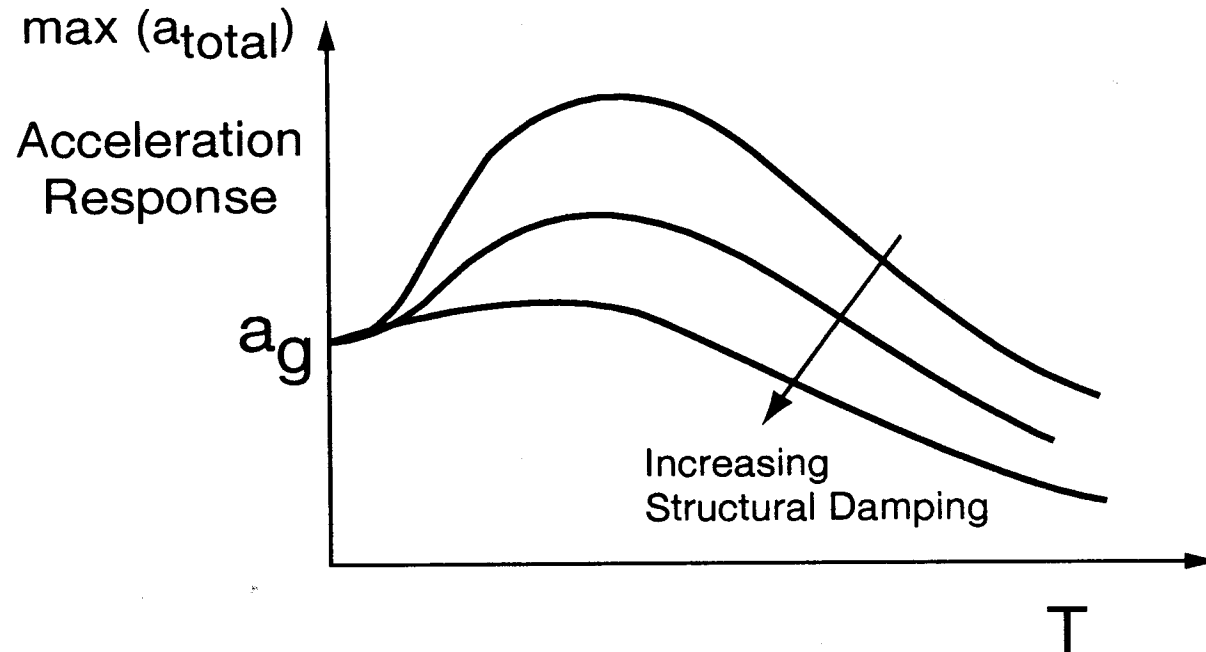
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# General Shape of Spectra

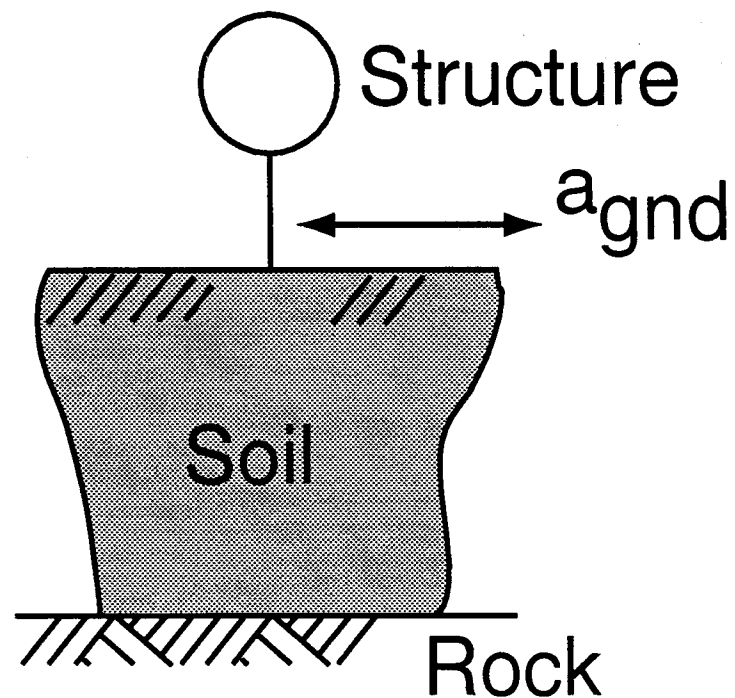


# Effects of Damping



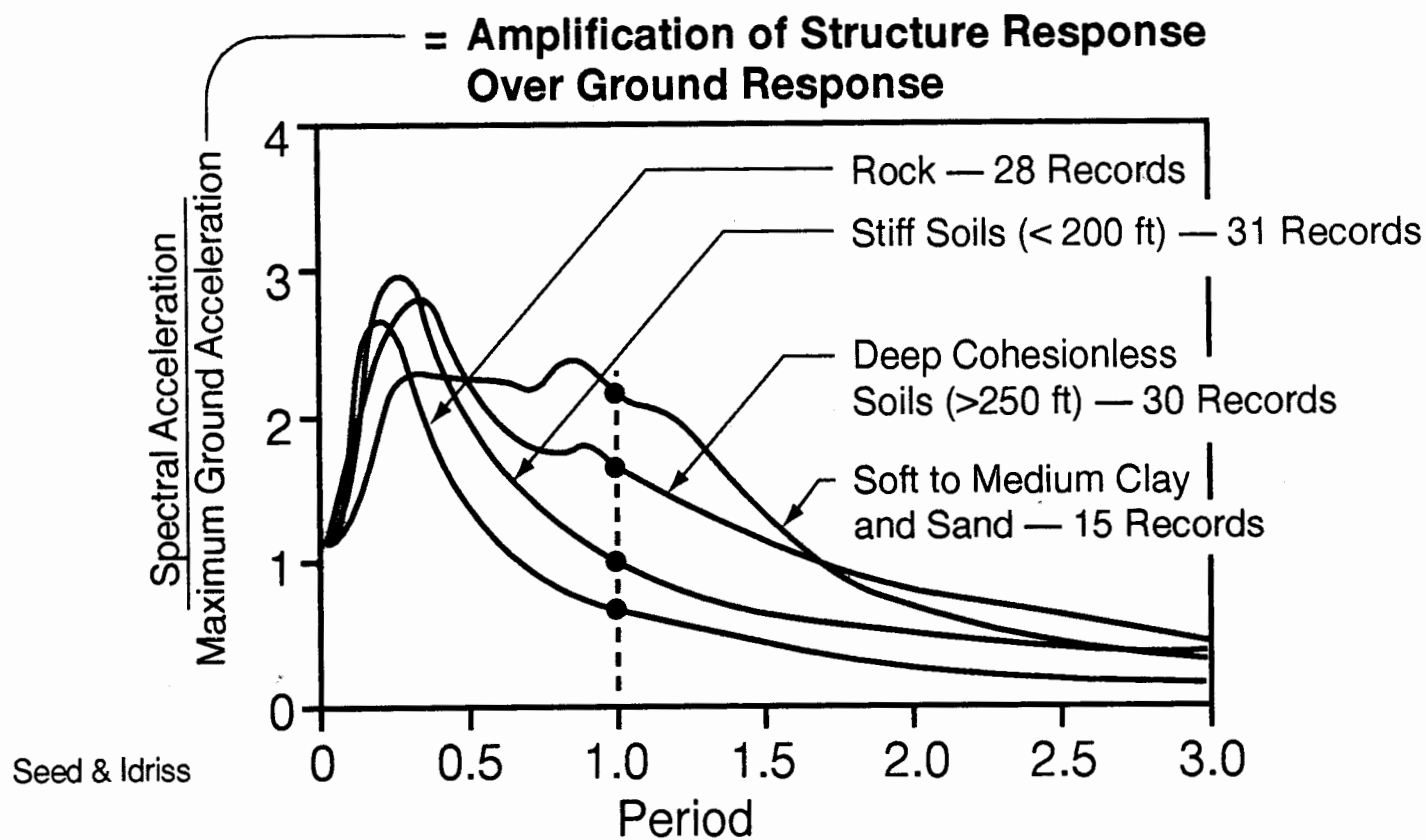
# Effects of Site Soil Conditions

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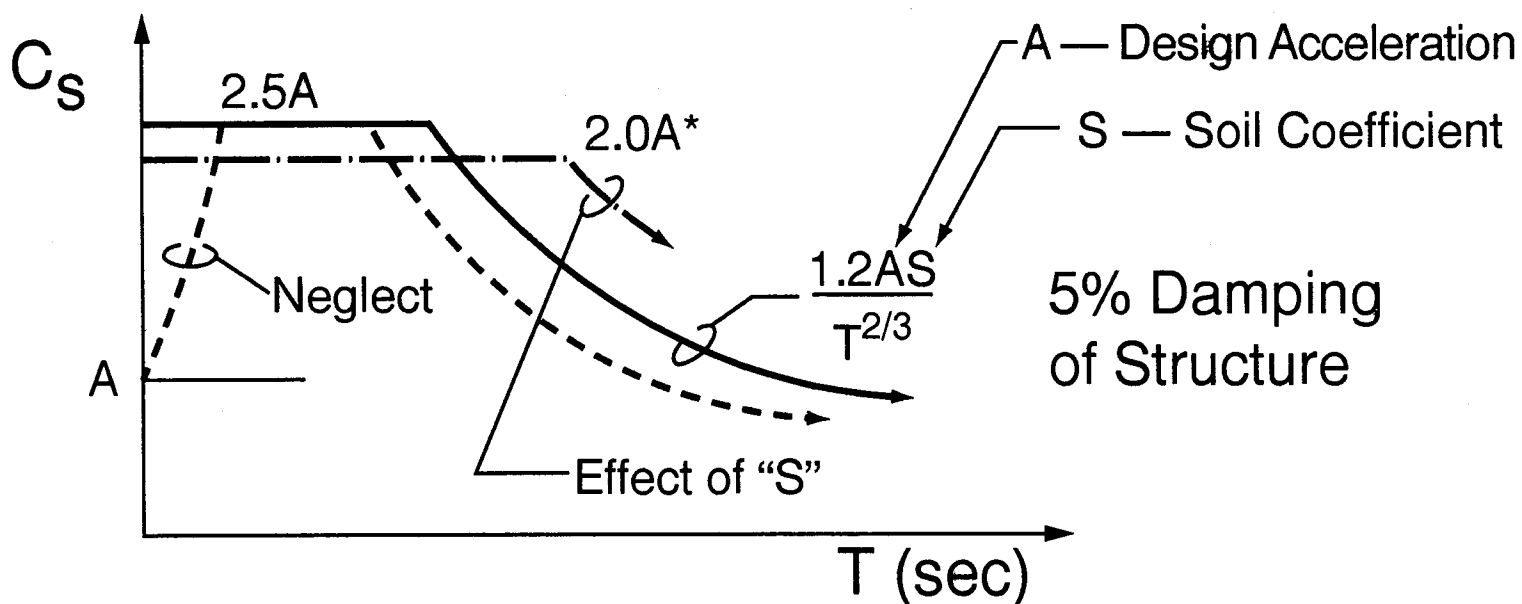




# Effects of Site Soil Conditions



# AASHTO Design Spectra



Total Acceleration or 'Spectral Acceleration'

$$a_{\text{total}} = C_s \cdot g$$

Acceleration Due to Gravity

\*  $2.0A - C_s$  Cap for Soft Soil when  $A \geq 0.30$

## Site Coefficient, S

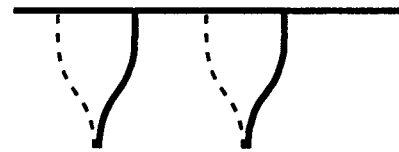
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**Table 2. Site Coefficient (S)**

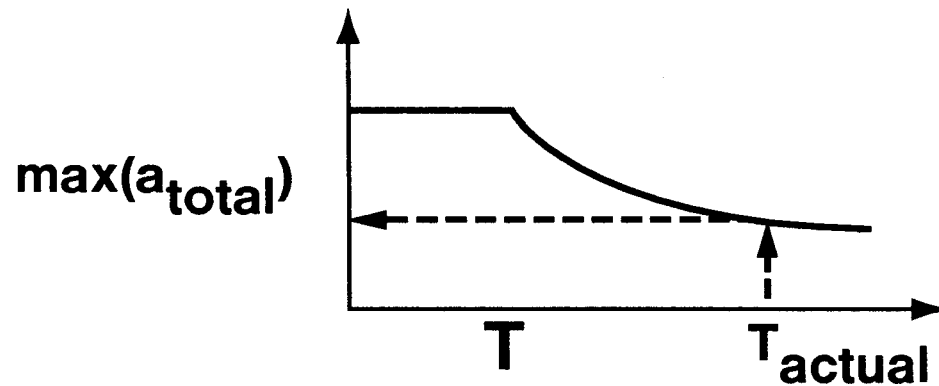
	Soil Profile Type			
	I	II	III	IV
S	1.0	1.2	1.5	2.0

- I. Rock or Stiff Soil < 200 ft Thick
- II. Deep Stiff Soil > 200 ft Thick
- III. Soft to Medium Clays and Sands > 30 ft Thick
- IV. Soft Clay or Silt > 40 ft Thick

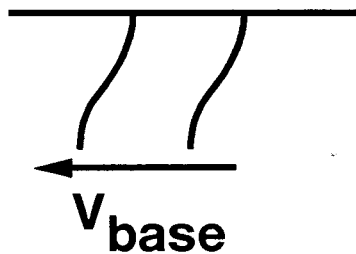
# How to Use a Response Spectrum



Determine Period of Bridge,  $T$



Use Spectrum to Find  $\max(a_{\text{total}})$



$$V_{\text{base}} = \text{Weighting Factor} \cdot \frac{W}{g} \cdot C_s$$

## **Session 2**

# **Overview of AASHTO Division 1-A**

---

- **Seismic Performance Category**
- **Choosing an Analysis Technique**
- **Response Modification Factors**
- **Overall Flow**

# Seismic Performance Categories, SPC

Seismic Hazard,

A from Map

Bridges  
Essential  
Other

IC  
I  
II

**TABLE 1. Seismic Performance Category (SPC)**

Acceleration Coefficient	Importance Classification (IC)	
A	I	II
$A \leq 0.09$	A	A
$0.09 < A \leq 0.19$	B	B
$0.19 < A \leq 0.29$	C	C
$0.29 < A$	D	C

SPC

# Design Requirements Tighten As Category Increases

SPC	Minimum Seat Widths	Seismic Analysis	Ductility Enhancing Details	Design for Plastic Hinging Forces	Approach Slabs
A	●				
B	●	●	●		
C	●	●	●	●	
D	●	●	●	●	●

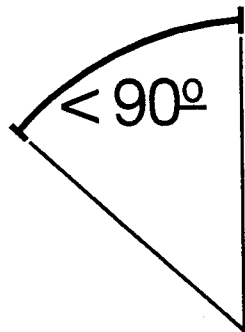
Diameter Proportional  
to Requirement Rigor

# Regular Bridges / 2 to 6 Spans

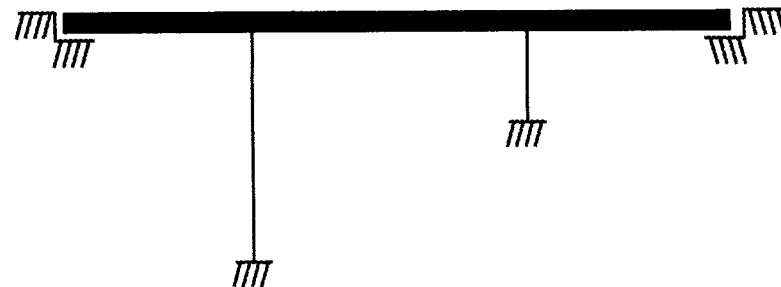
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Span Length Ratio



Curvature in Plan



Pier Stiffness Ratio



# Minimum Analysis Requirements

---

**TABLE 4. Minimum Analysis Requirements**

<b>Seismic Performance Category</b>	<b>Regular Bridges with 2 Through 6 Spans</b>	<b>Not Regular Bridges with 2 or More Spans</b>
A	Not required	Not required
B, C, D	Use procedure 1 or 2	Use procedure 3

# Seismic Analysis Procedures

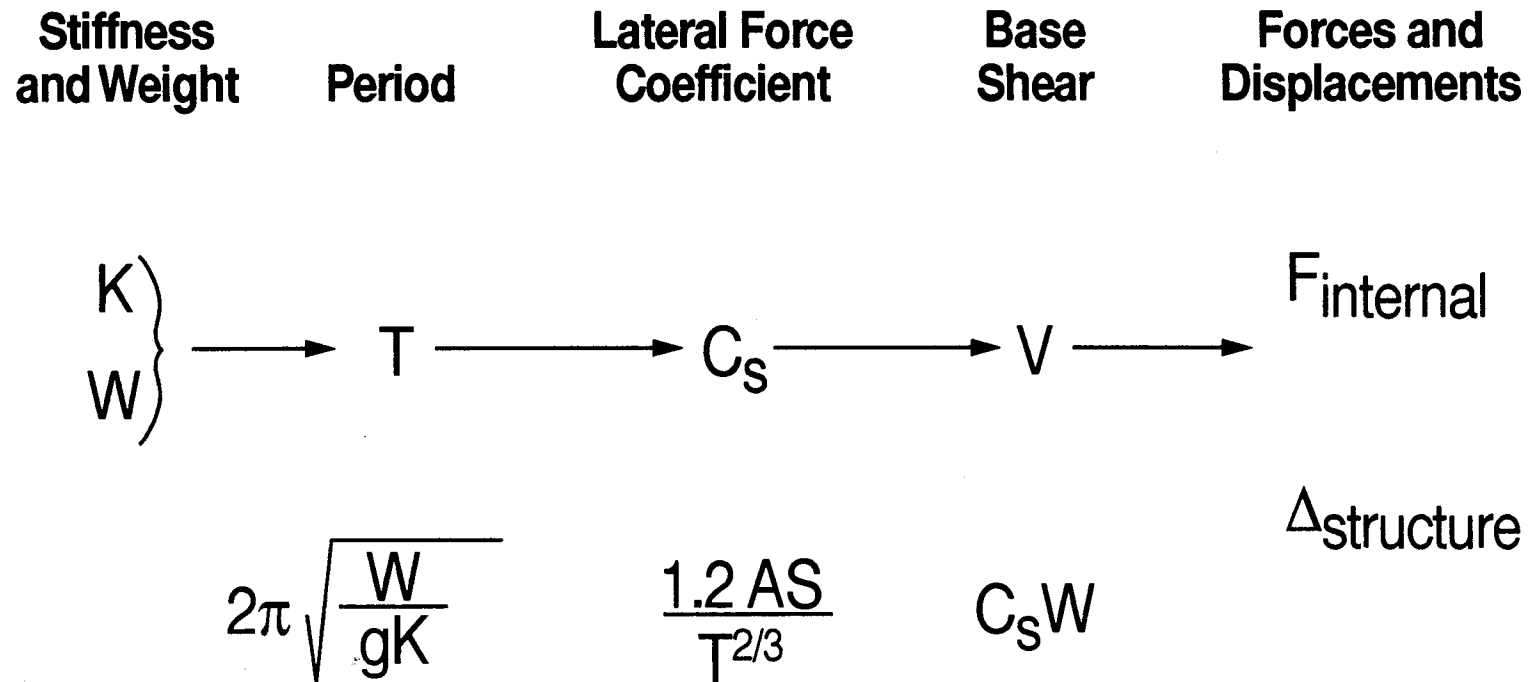
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1. Uniform Load Method
2. Single-Mode Spectral Method
3. Multimode Spectral Method
4. Time History Method



# Flow of Analysis Procedures

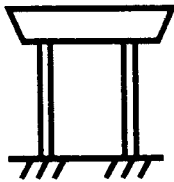
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# Response Modification Factor, R

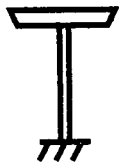
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$$M_{\text{design}} = M_{\text{elastic}}/R$$



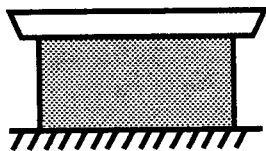
High Redundancy,  
Ductile

$$R = 5$$



No Redundancy,  
Ductile

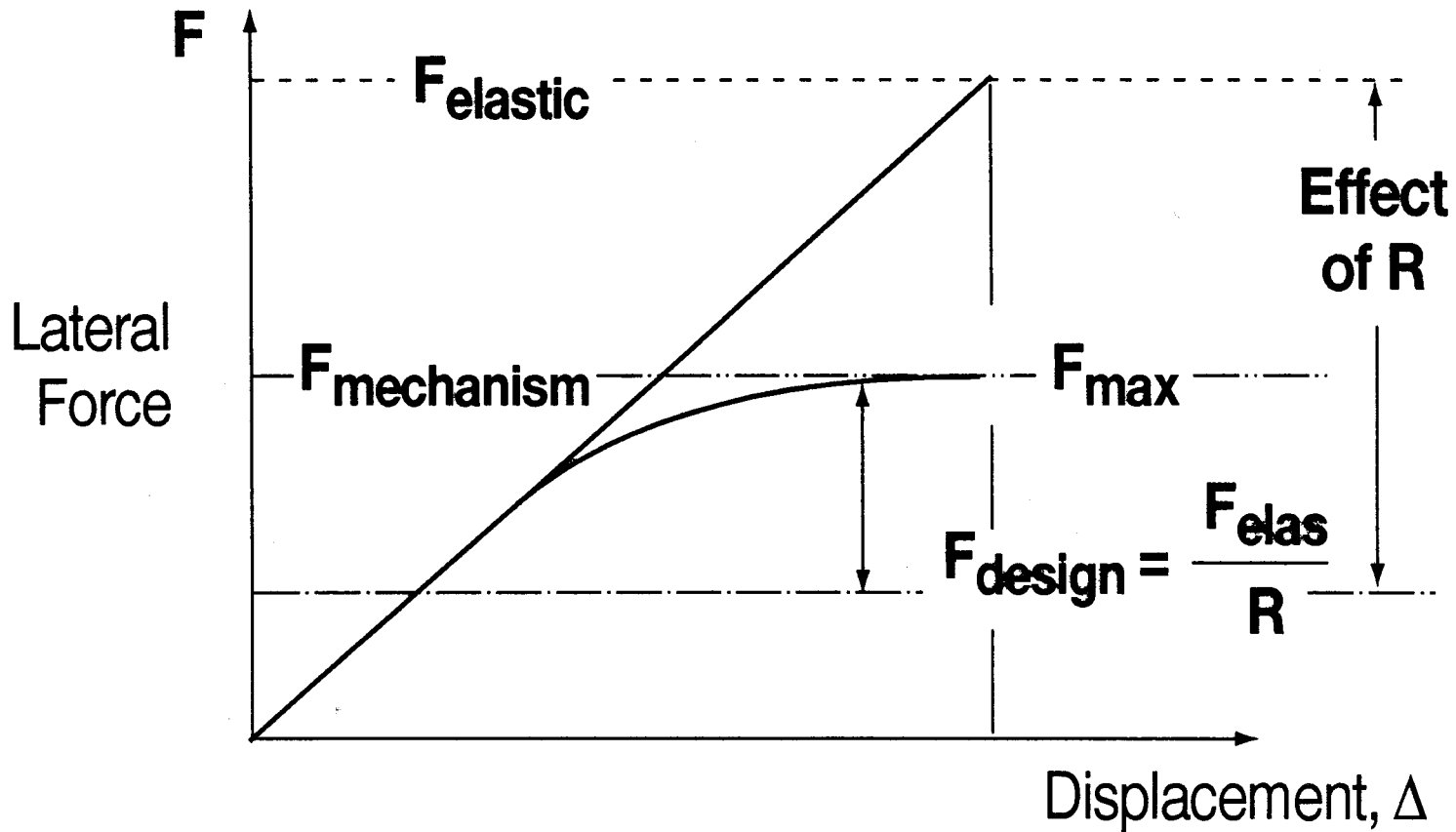
$$R = 3$$



Limited Ductility

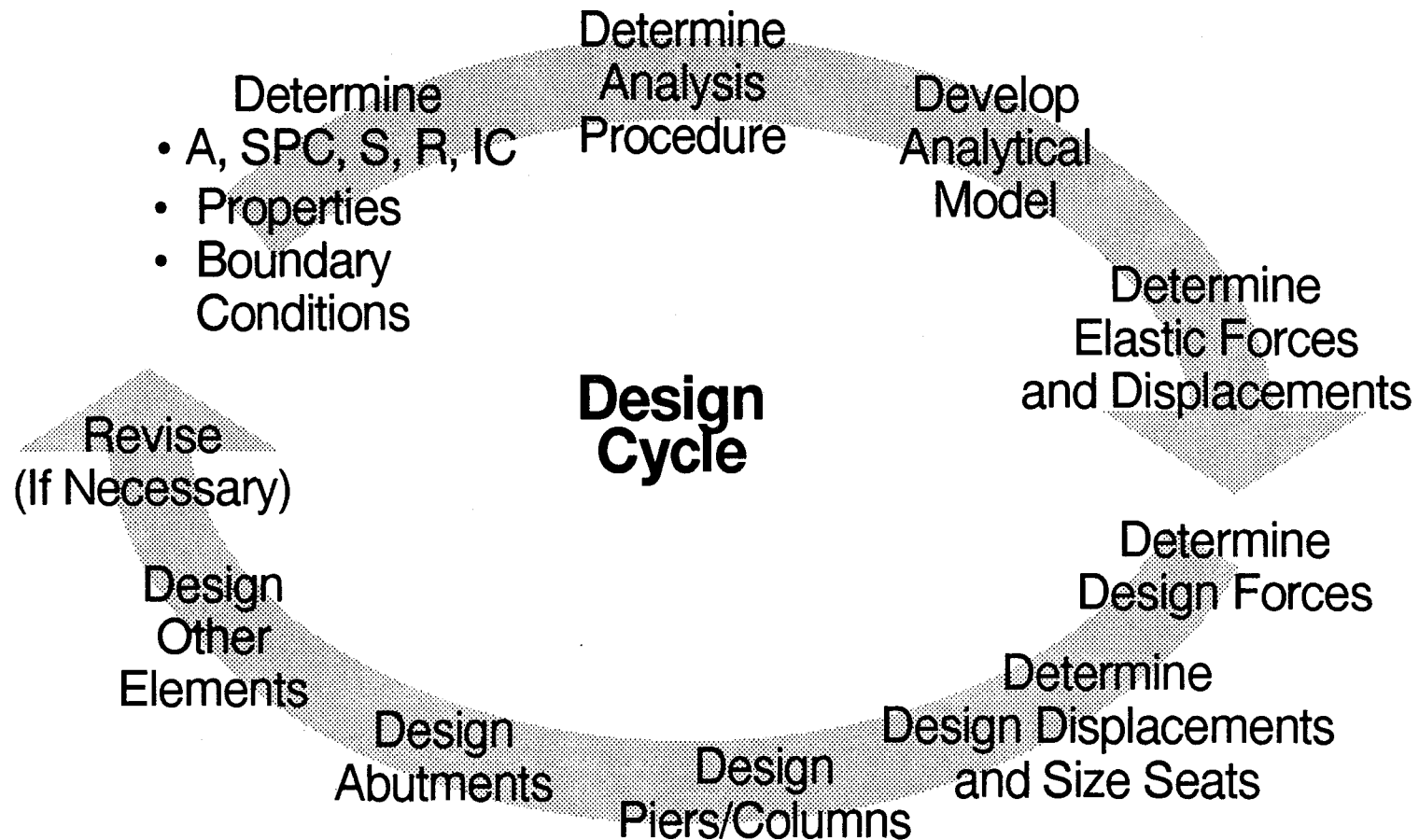
$$R = 2$$

# Elastic vs. Design vs. Actual Forces



# AASHTO Division 1-A

---





# **Session 3**

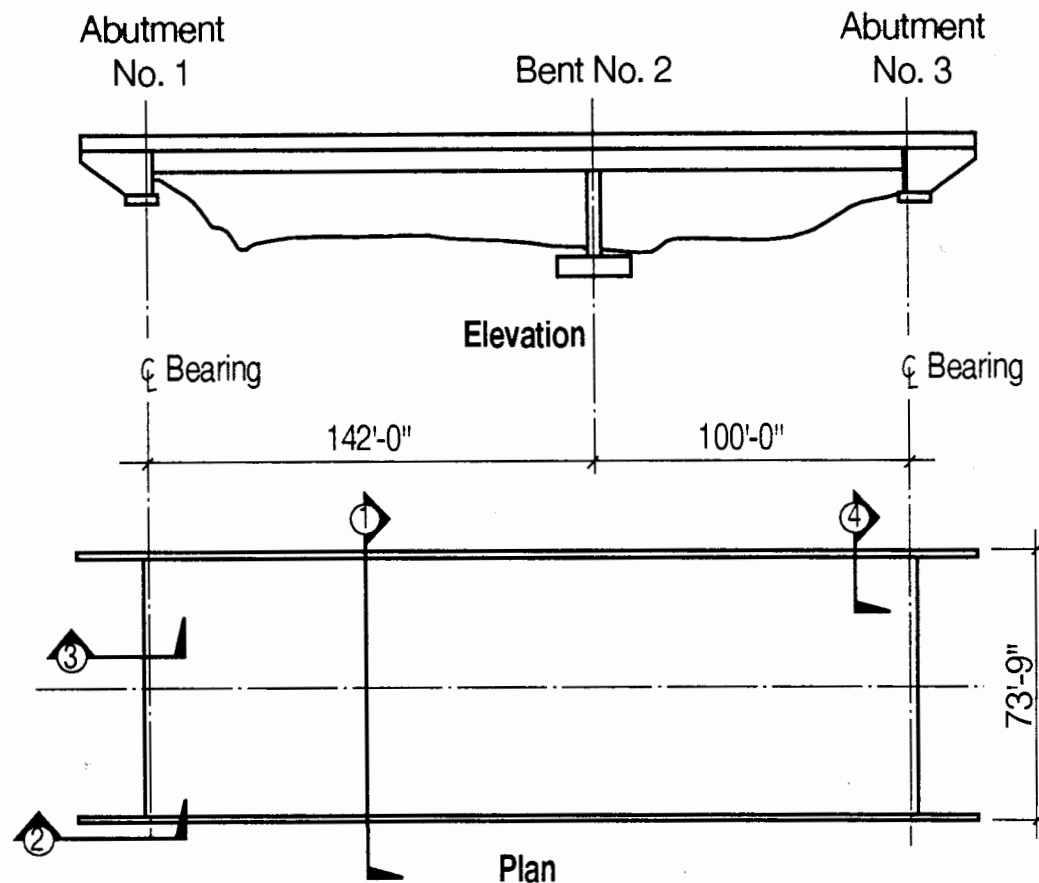
## **Example Analysis / Two-Span Bridge**

---

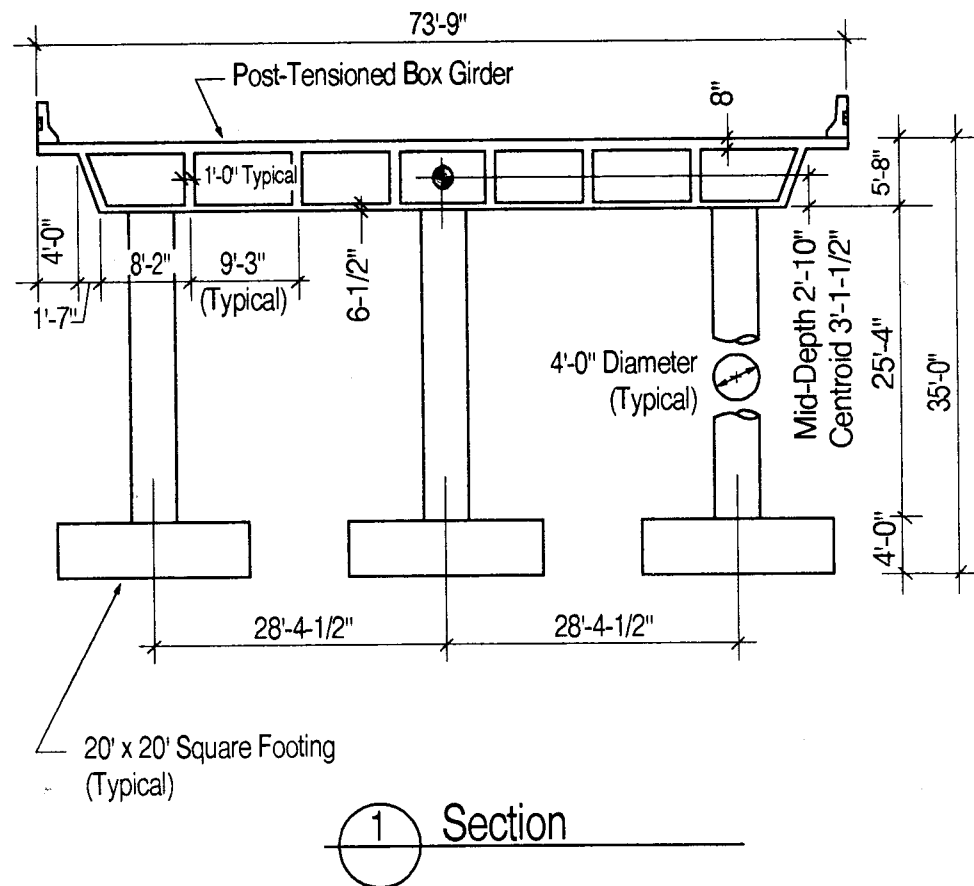
- **Bridge Layout and Basic Data**
- **Behavior**
- **Mathematical Model**
- **Earthquake Direction**
- **Longitudinal Analysis**
- **Transverse Analysis**



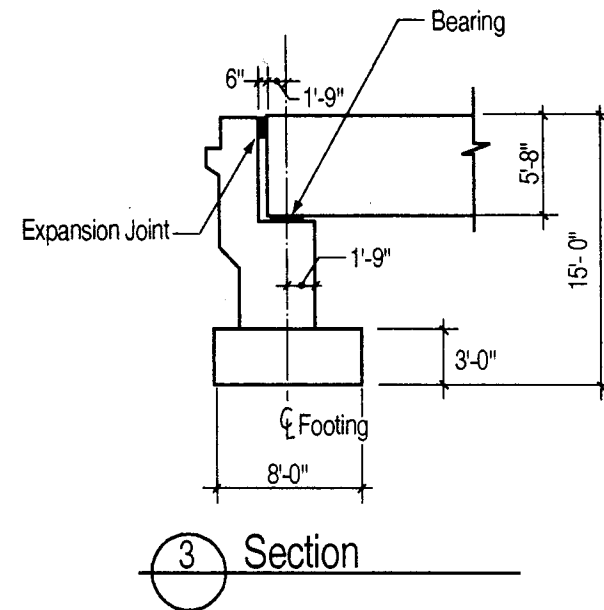
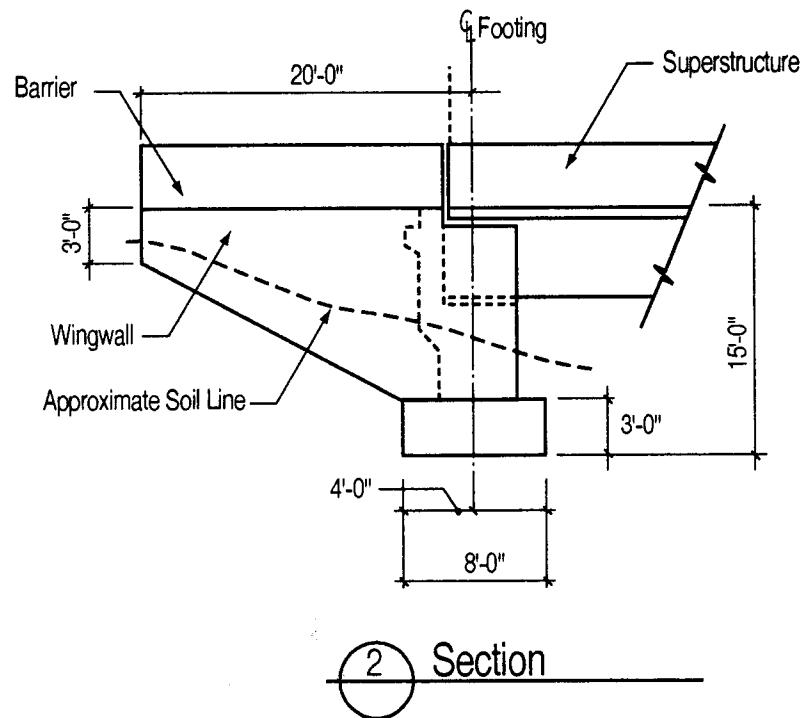
# Layout / Plan and Elevation



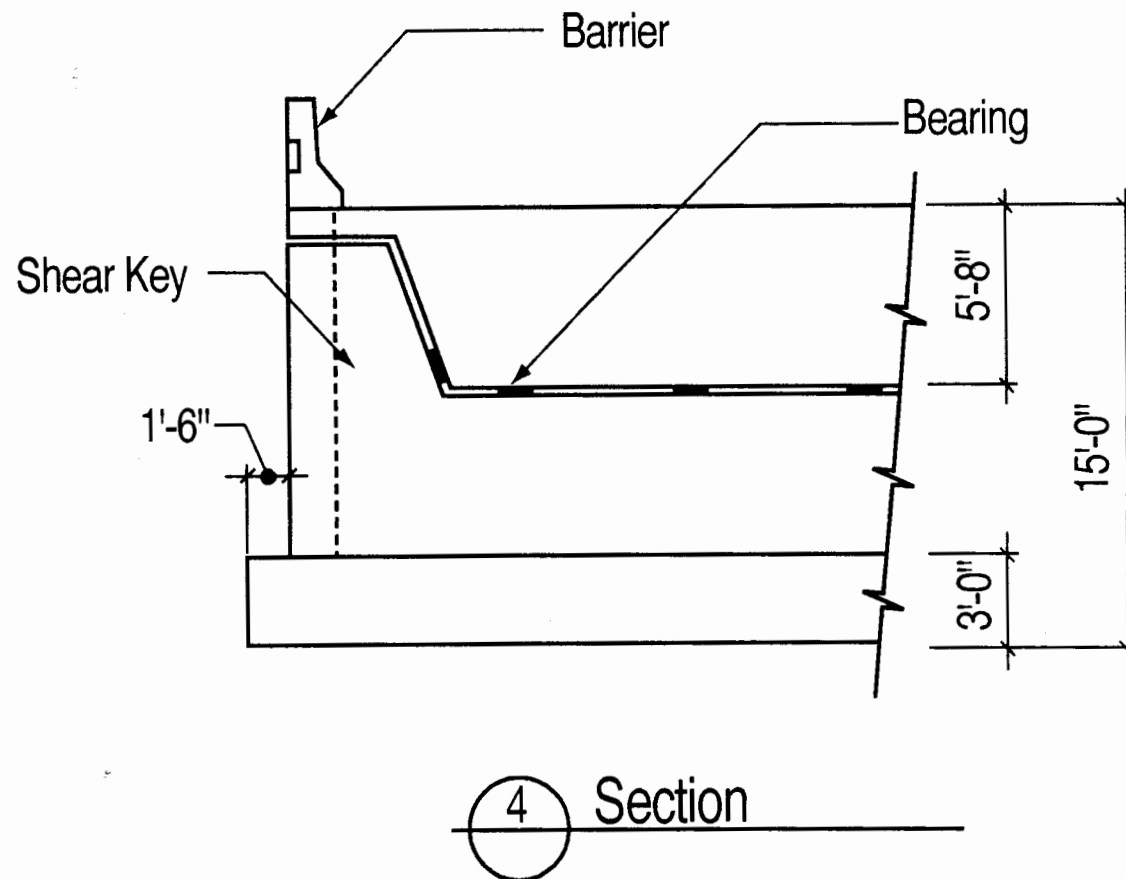
# Layout / Preliminary Bent Details



# Bridge Layout / Abutment Details



# Layout / Shear Key at Abutments



## Basic Data for Bridge

---

- From AASHTO Map: **A = 0.28g**  
(Interpolation Permitted)
- Soil Is **250 ft Deep Glacial Sand and Gravel**
- Bridge Is **Not Essential**

# Determine Seismic Performance Category

$A = 0.28 \text{ g}$      $IC = \text{Not Essential}$



**TABLE 1. Seismic Performance Category (SPC)**

Acceleration Coefficient	Importance Classification (IC)	
	I	II
$A \leq 0.09$	A	A
$0.09 < A \leq 0.19$	B	B
$0.19 < A \leq 0.29$	C	C
$0.29 < A$	D	C

SPC C

# Determine Soil Site Coefficient

Four Basic Types of Soil:

- I. Rock or Stiff Soil < 200 ft Thick
- II. Deep Stiff Soil > 200 ft Thick**
- III. Soft to Medium Clay and Sands > 30 ft Thick
- IV. Soft Clay or Silt > 40 ft Thick

**TABLE 2. Site Coefficient**

Soil Profile Type				
	I	II	III	IV
S	1.0	1.2	1.5	2.0

$S = 1.2$

# Response Modification Factors

## Intermediate Substructure = Multiple Column Bent

**TABLE 3. Response Modification Factor (R)**

Substructure	R	Connections	R
Wall-Type Pier	2	Superstructure to Abutment	0.8
Reinforced Concrete Pile Bents		Expansion Joints within a	
a. Vertical Piles Only	3	Span of the Superstructure	0.8
b. One or More Batter Piles	2	Columns, Piers, or Pile Bents	
Single Columns	3	to Cap Beam or Superstructure	1.0
Steel or Composite Steel		Columns or Piers to Foundations	1.0
and Concrete Pile Bents			
a. Vertical Piles Only	5		
b. One or More Batter Piles	3		
Multiple Column Bent	5		



# Determine Analysis Procedure

- Straight Alignment
- Span Length Ratio:  
 $\frac{142}{100} = 1.42 < 3$
- Bent Stiffness Ratio: NA



**TABLE 5. Regular Bridge Requirements**

Parameter	Value				
Number of Spans	2	3	4	5	6
Maximum Subtended Angle (Curved Bridge)	90°✓	90°	90°	90°	90°
Maximum Span Length Ratio from Span-to-Span	3 ✓	2	2	1.5	1.5
Maximum Bent/Pier Stiffness Ratio from Span-to-Span (Excluding Abutments)	-- ✓ ↓	4	4	3	2

Regular

# Determine Analysis Procedure (continued)

**TABLE 5. Minimum Analysis Requirements**

Seismic Performance Category	Regular Bridges with 2 Through 6 Spans	Not Regular Bridges with 2 or More Spans
A	Not Required	Not Required
B, C, D	Use Procedure 1 or 2	Use Procedure 3

May Use:

1. Uniform Method  
Longitudinal
2. Single-Mode Spectral  
Transverse
3. Multimode Spectral
4. Time History

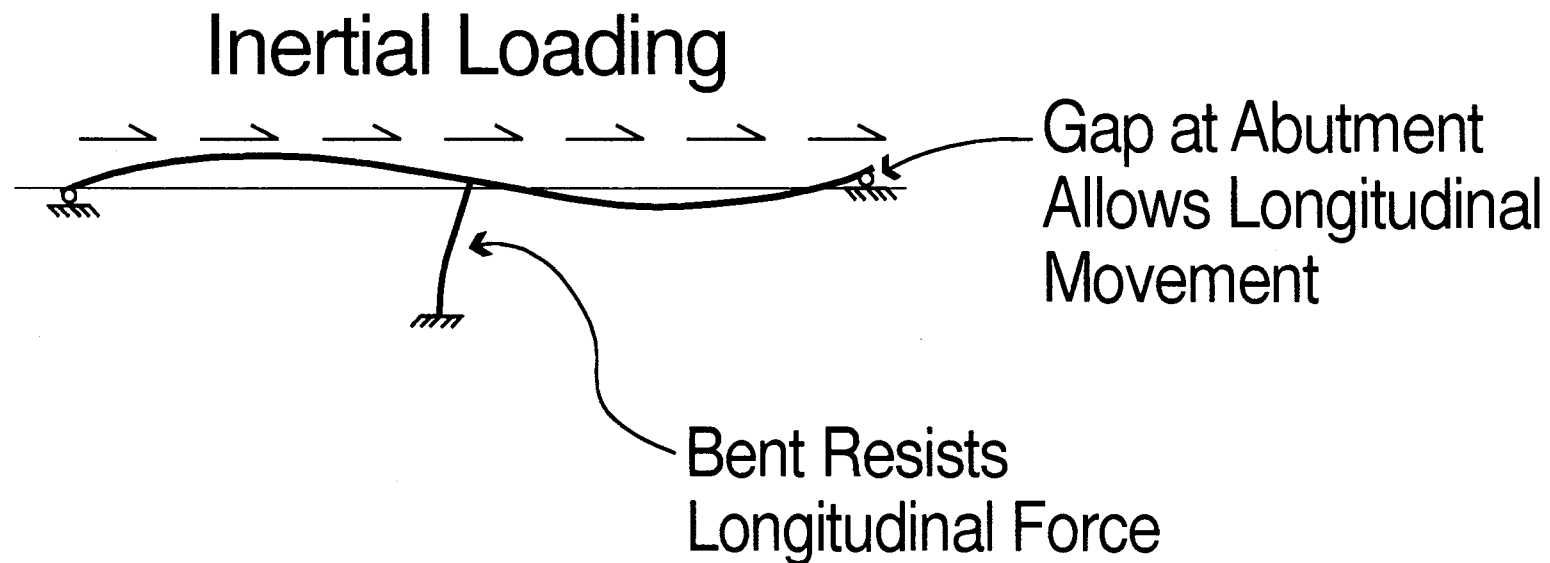
## Session 3

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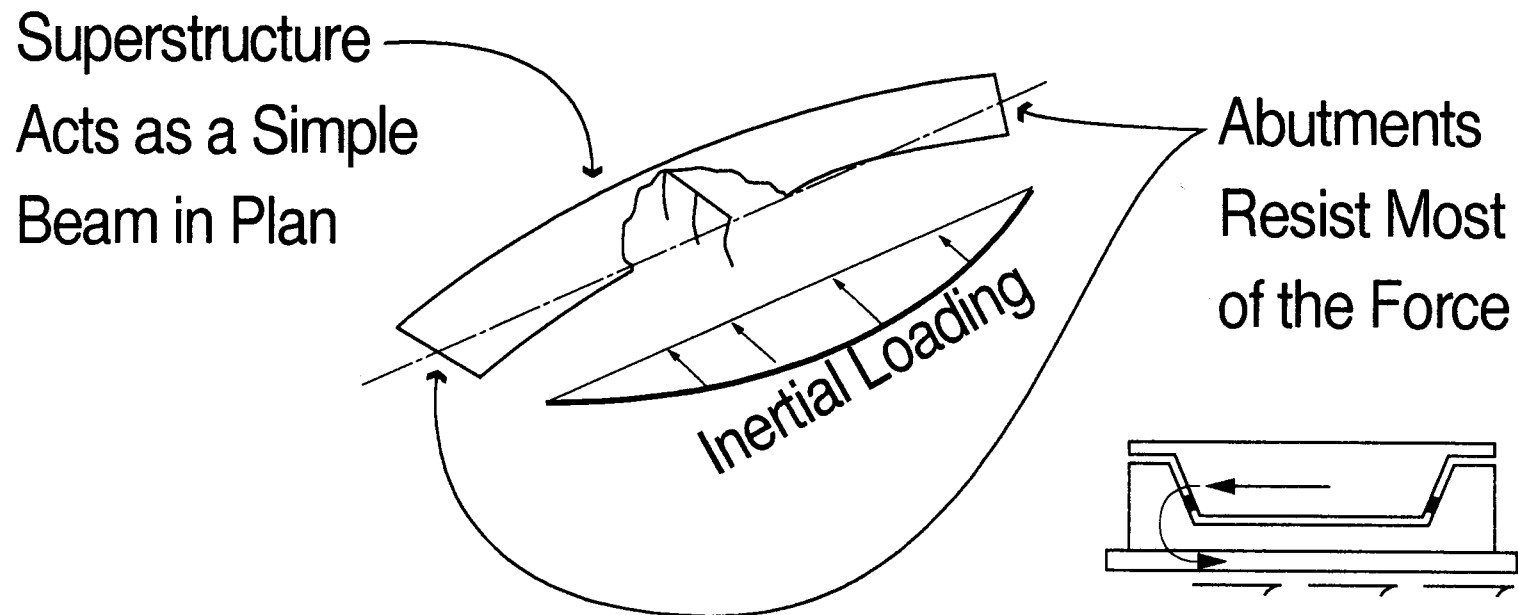
- Bridge Layout and Design
- **Behavior**
- Mathematical Model
- Earthquake Direction
- Longitudinal Analysis
- Transverse Analysis

# Longitudinal Lateral Load Behavior

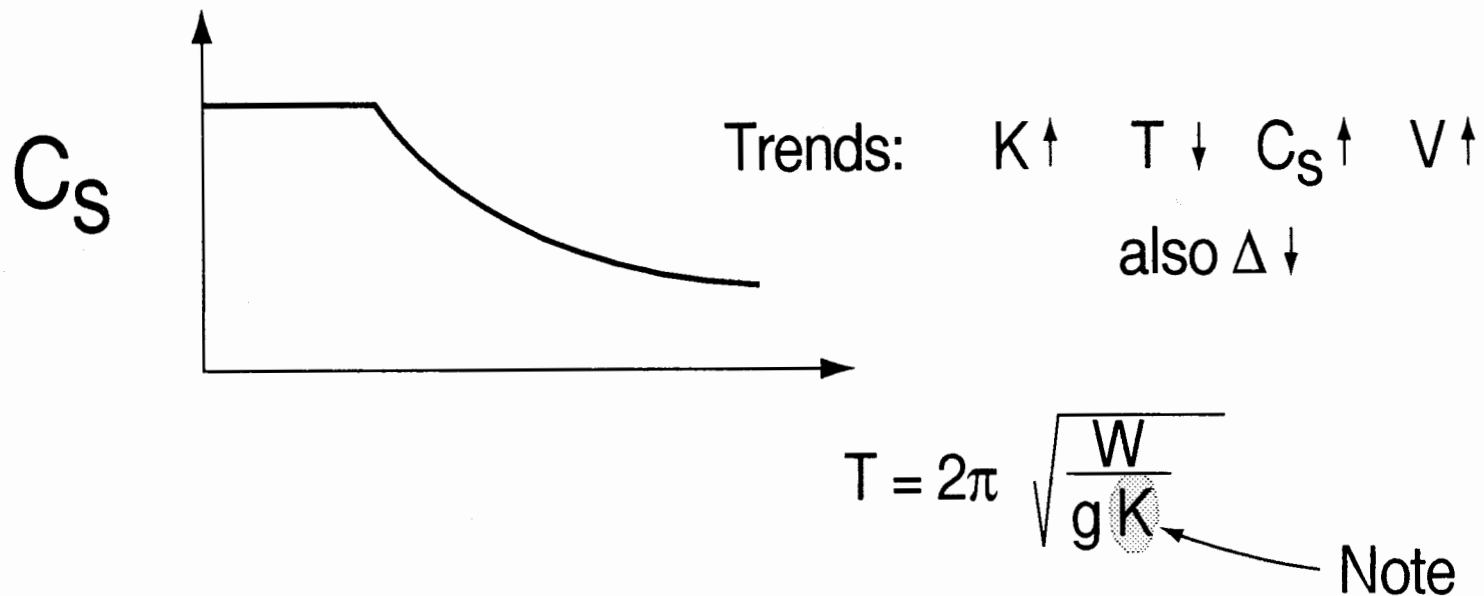
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# Transverse Lateral Load Behavior



## Bounding the Response



- **Total Base Shear:**  $V$  Is Proportional to  $C_s \cdot W$

# Bounding the Response (continued)

---

## Implications

In General, Stiffening the Structure Leads to

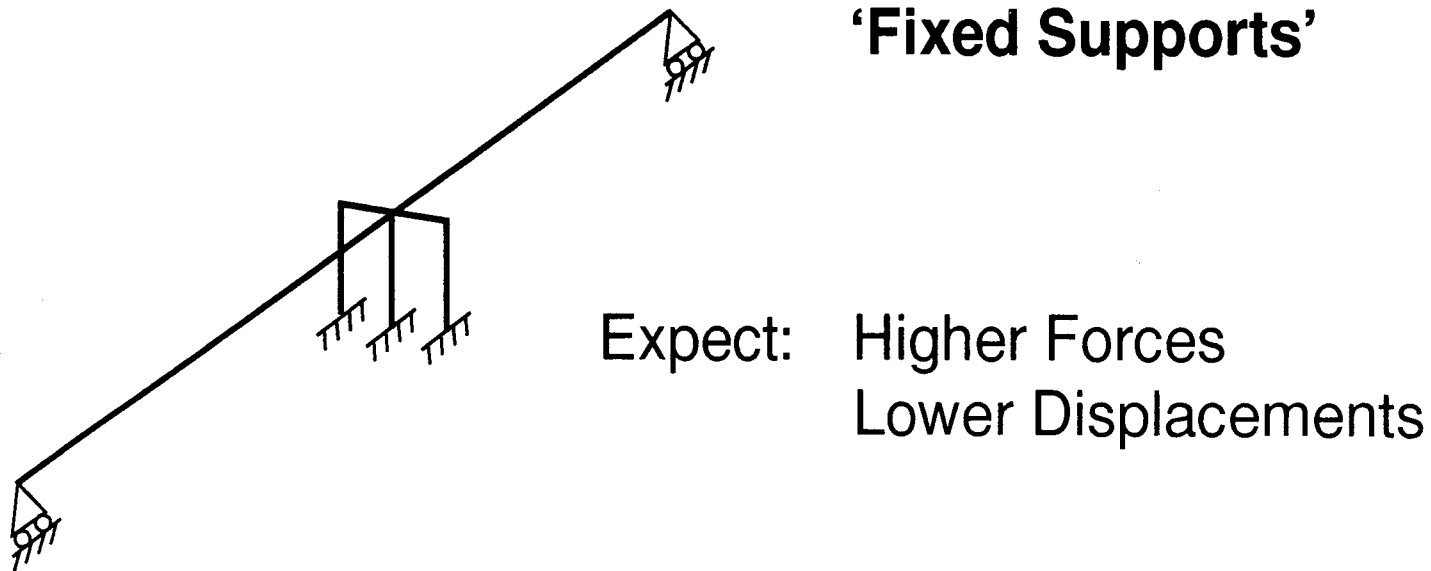
- Larger Forces
- Smaller Displacements

Conversely, Softening the Structure Leads to

- Smaller Forces
- Larger Displacements

# Alternatives to Consider / No. 1

---

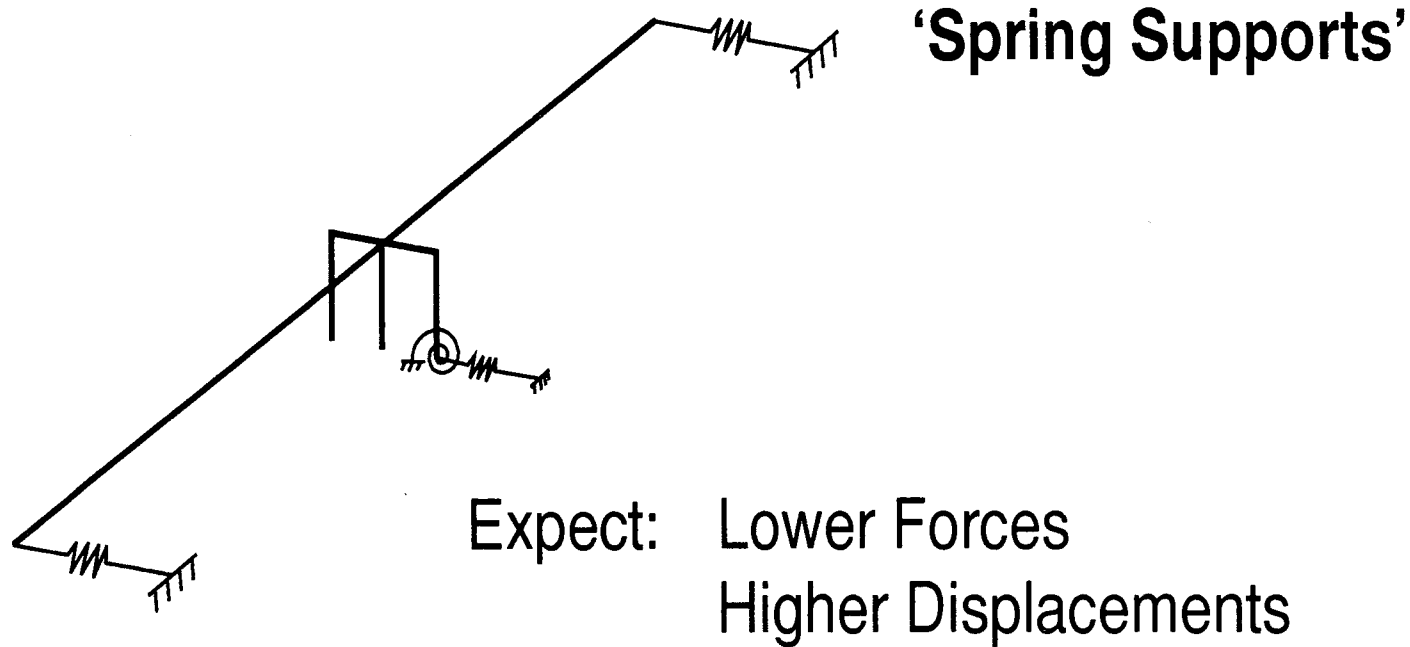


Use for Analysis to Get  
Upper-Bound for Elastic Forces



## Alternatives to Consider / No. 2

---

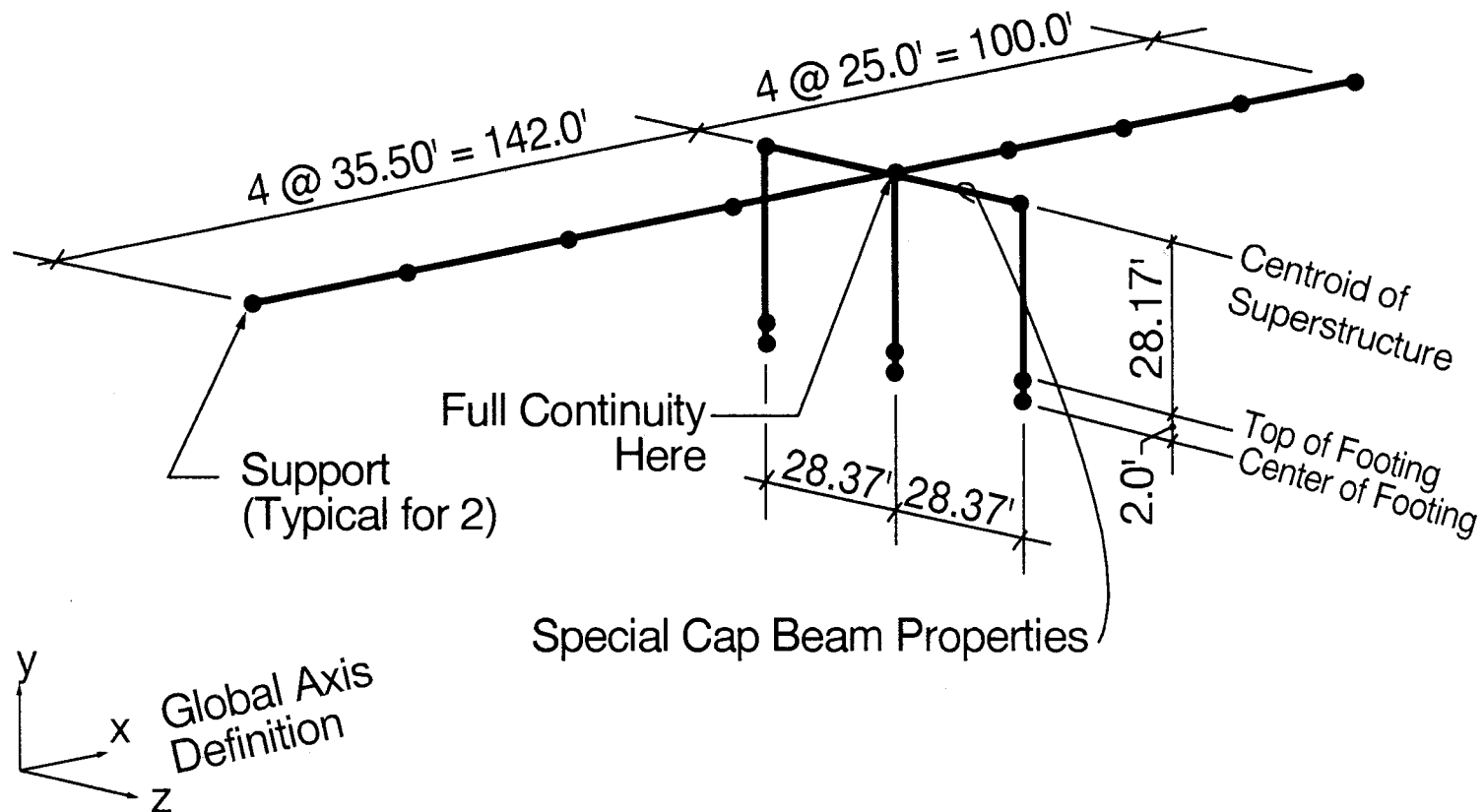


## Session 3

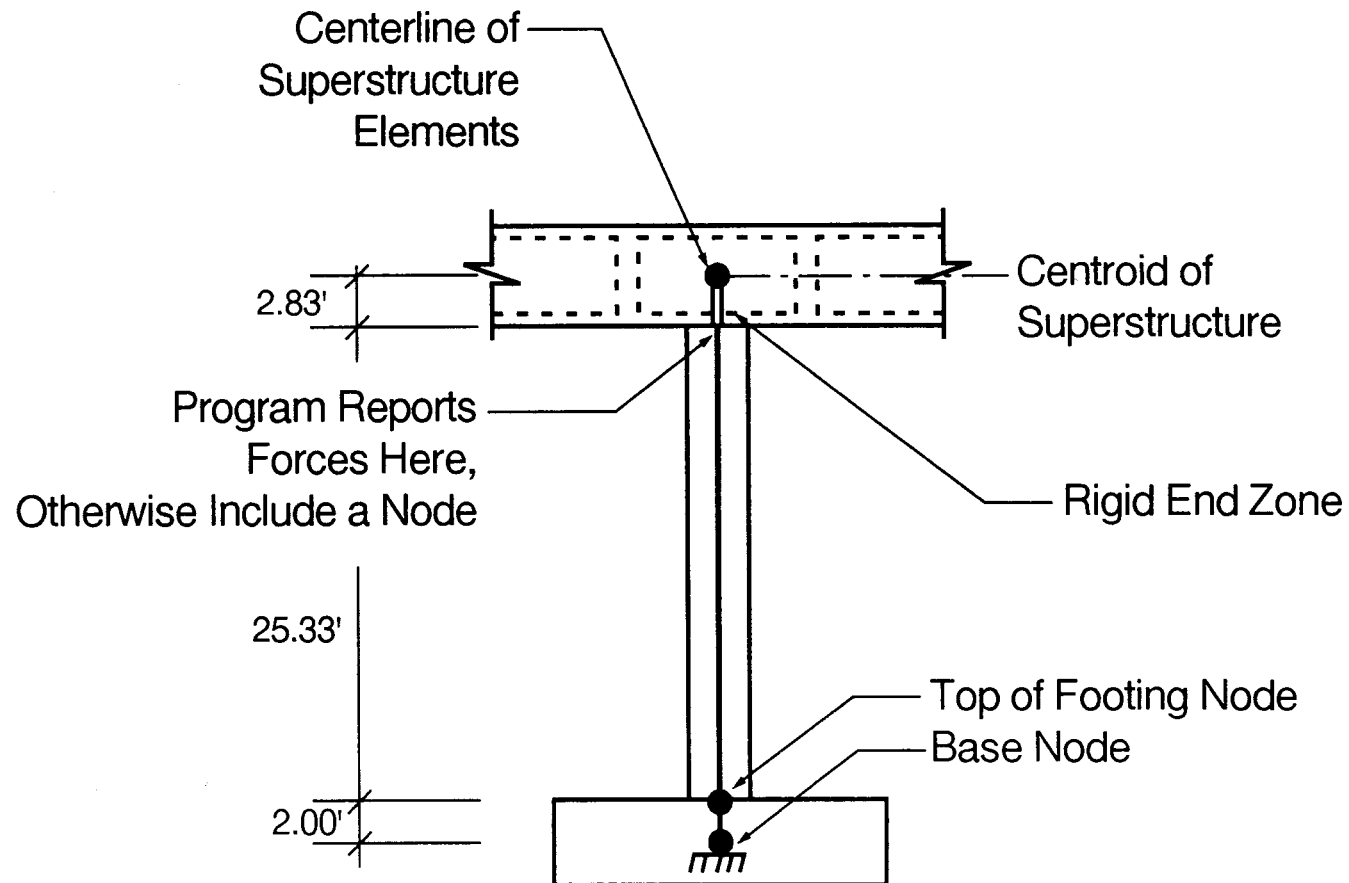
---

- Bridge Layout and Design
- Behavior
- **Mathematical Model**
- Earthquake Direction
- Longitudinal Analysis
- Transverse Analysis

# Mathematical Model for Analysis

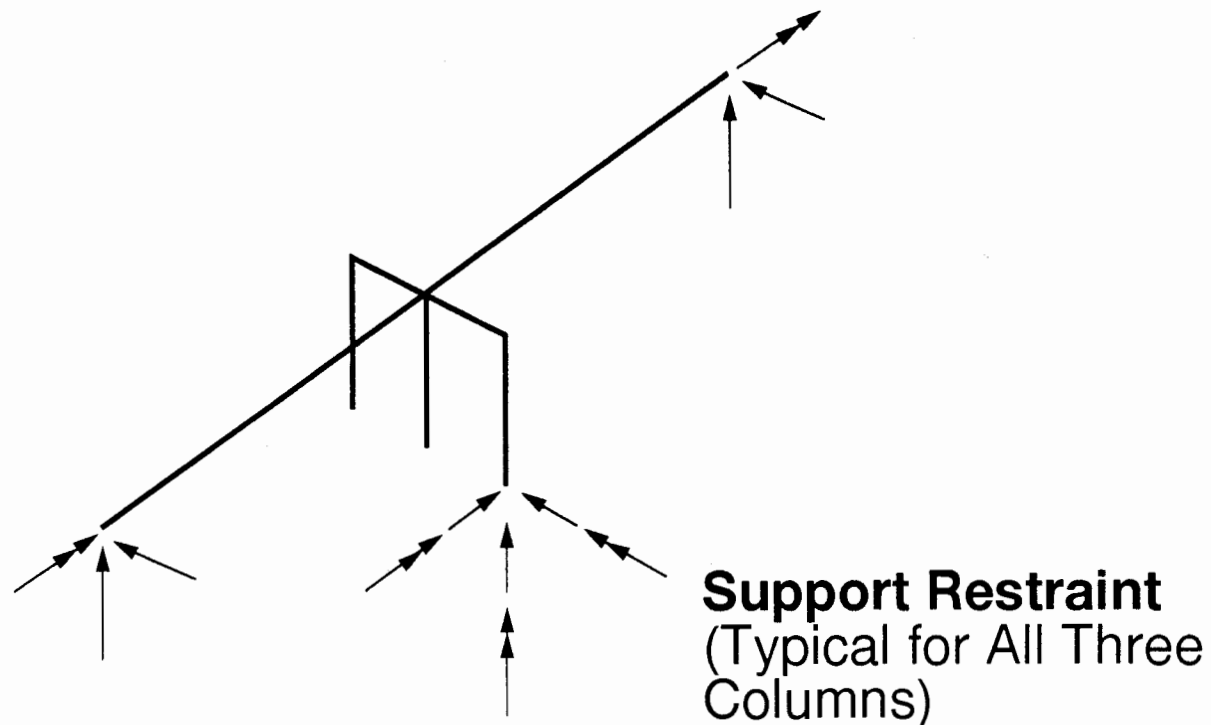


# Column and Footing Element Geometry



# Support Conditions

---



Vector Arrows Indicate Support Restraint in the Direction Shown

# Properties

---

$$f'_c = 4000 \text{ psi} \quad E = 518,400 \text{ ksf}$$

## Superstructure

$$A = 120 \text{ ft}^2$$

$$I_{\text{str}} = 51,000 \text{ ft}^4$$

$$I_{\text{weak}} = 575 \text{ ft}^4$$

## Column

$$A = 12.6 \text{ ft}^2$$

$$I = 12.6 \text{ ft}^4$$

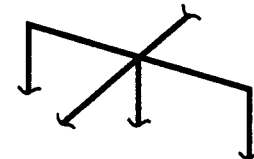
## Cap Beam

$$A = 25 \text{ ft}^2$$

$$I_{\text{str}} = 10^7 \text{ ft}^4$$

$$I_{\text{weak}} = 10^7 \text{ ft}^4$$

Properties for Lateral Analysis Only



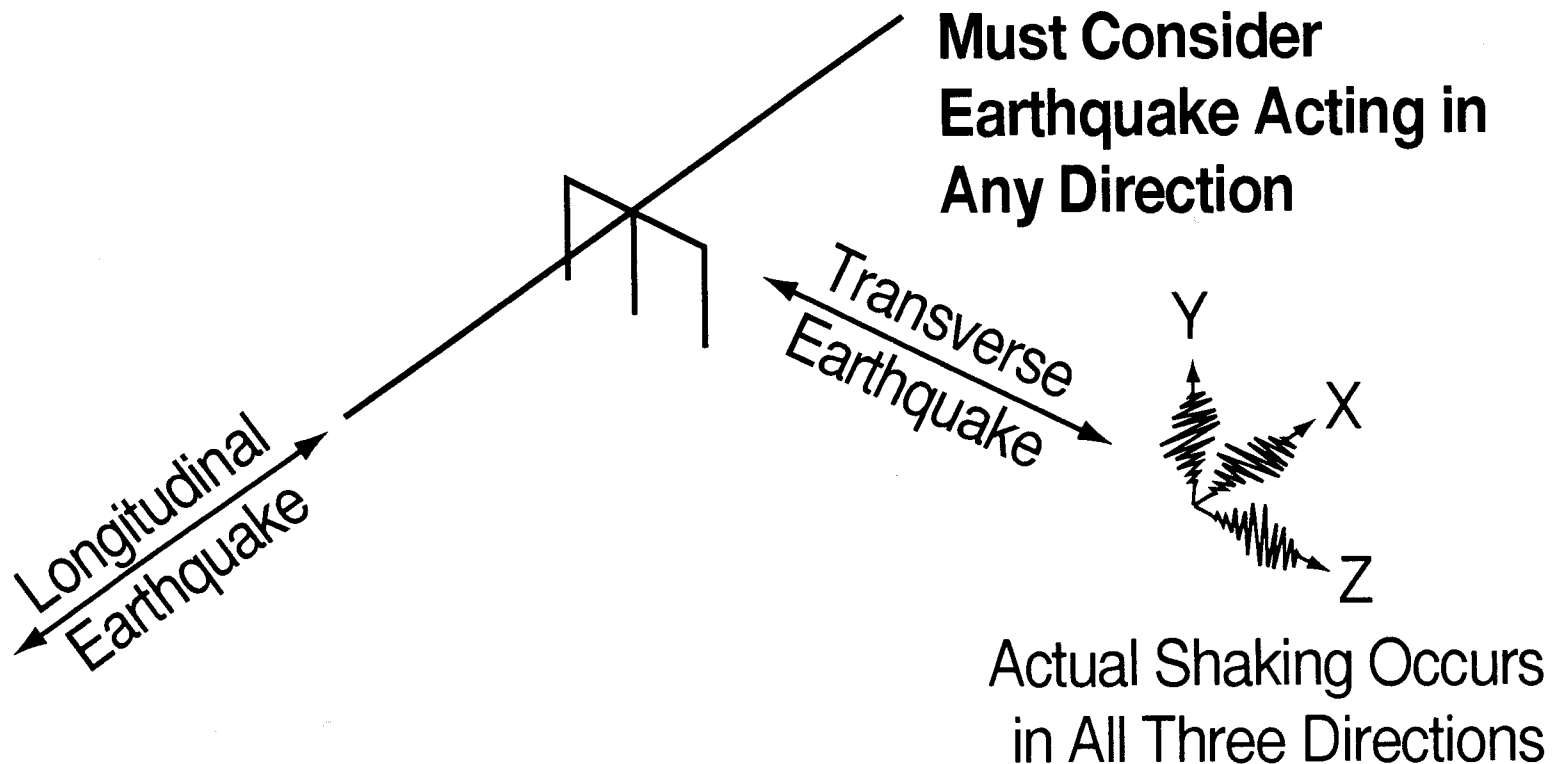
## Session 3

---

- Bridge Layout and Design
- Behavior
- Mathematical Model
- **Earthquake Direction**
- Longitudinal Analysis
- Transverse Analysis

# Earthquake Direction

---





# Directional Combinations for Loading

---

## Two Analyses:

- Orthogonal Horizontal Directions
- Actual Earthquake Attack May Be from Any Direction
- Maximum Inputs **Do Not** Occur Simultaneously in Each Direction

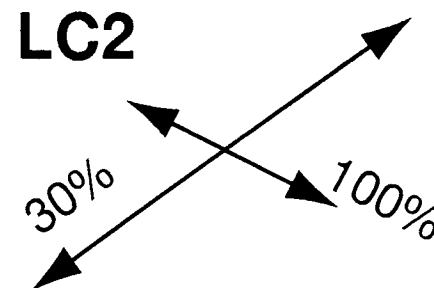
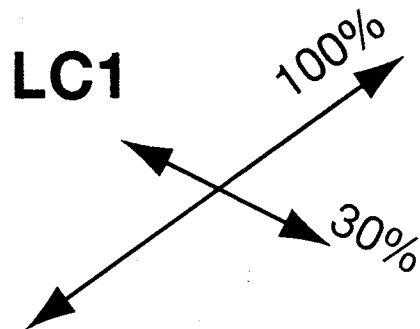
# Directional Combinations for Loading (continued)

---

- Load Combinations**

LC1 = 100% Longitudinal + 30% Transverse

LC2 = 100% Transverse + 30% Longitudinal



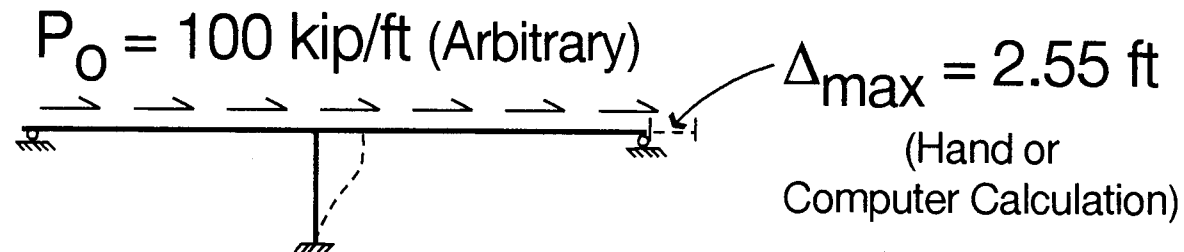
## Session 3

---

- Bridge Layout and Design
- Behavior
- Mathematical Model
- Earthquake Direction
- **Longitudinal Analysis**
- Transverse Analysis

## Uniform Load Method / Section 4.3 (1-A)

- **Step 1**



- **Step 2**

$$\text{Stiffness } K = \frac{P_O L}{\Delta_{\max}} = \frac{100(242)}{2.55} = 9486 \text{ kip/ft}$$

$$\text{Weight } W = 4876 \text{ kip}$$

- **Step 3**

$$T = 2\pi \sqrt{\frac{W}{gK}} = 2\pi \sqrt{\frac{4876}{32.2 (9486)}} = 0.79 \text{ sec}$$

## Uniform Load Method (continued)

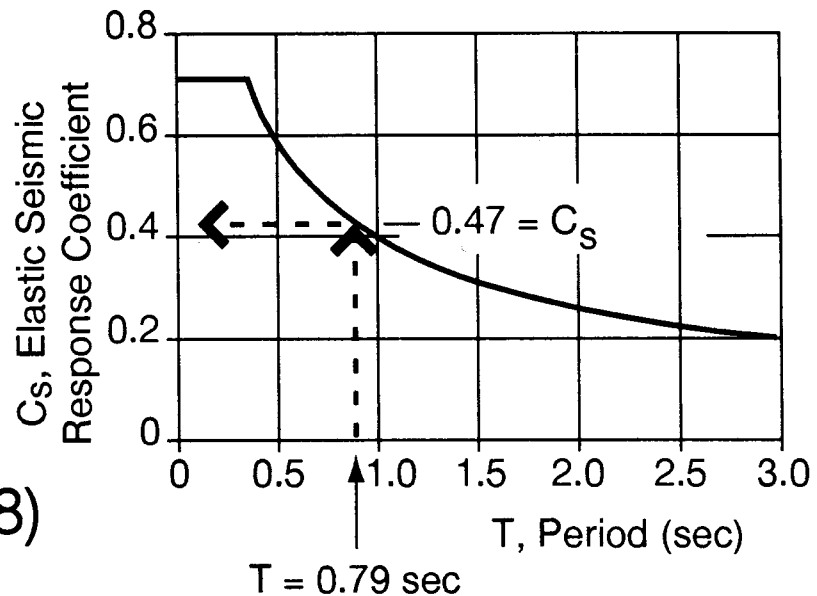
### • Step 4

$$C_S = \frac{1.2AS}{T^{2/3}} \leq 2.5A$$

$$C_S = \frac{1.2(0.28)1.2}{(0.79)^{2/3}} \leq 2.5(0.28)$$

$$C_S = 0.47 \leq 0.70$$

Controls



## Uniform Load Method (continued)

---

### • Step 4 (continued)

$$P_e(x) = \frac{C_s W}{L} = \frac{0.47 (4876)}{242}$$

$$P_e(x) = 9.47 \text{ kip/ft}$$

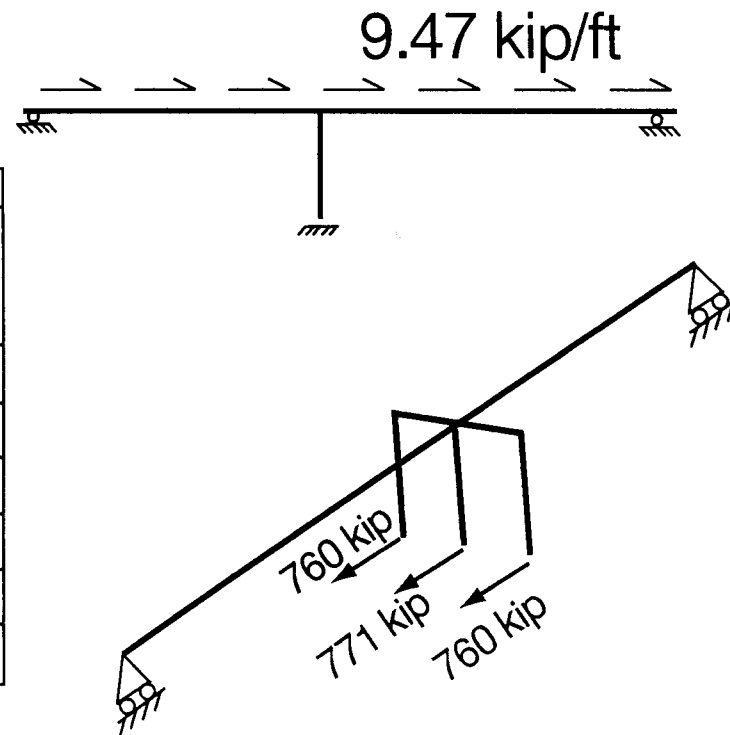
Earthquake Load



# Uniform Load Method (continued)

## • Step 5

			Forces and Moments				
			Longitudinal		Transverse		Axial (kips)
			Shear (kips)	Moment (kip-ft)	Shear (kips)	Moment (kip-ft)	
Abutment No. 1			0	0	0	0	105
Bent No. 2	Center	Top	771	9978	0	0	35.5
		Bottom	771	9566	0	0	35.5
	Outboard	Top	760	9790	0	0	35.5
		Bottom	760	9481	0	0	35.5
Abutment No. 3			0	0	0	0	211



# Session 3

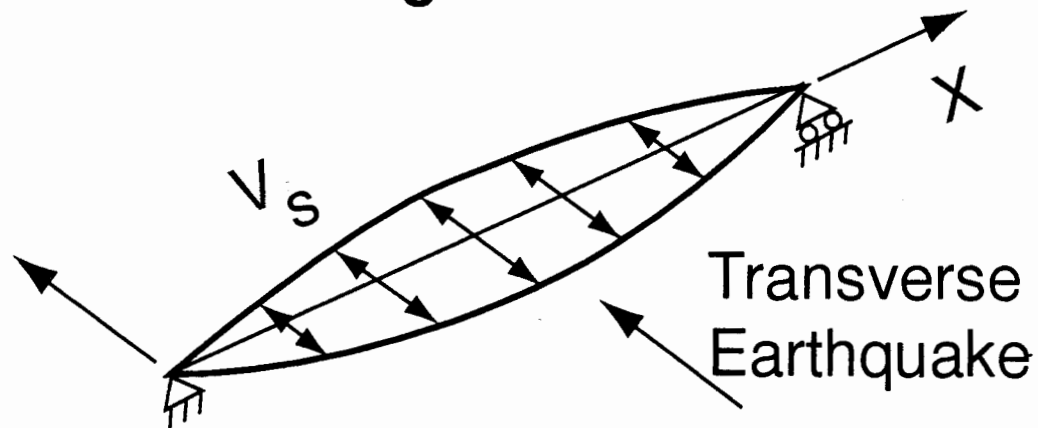
---

- Bridge Layout and Design
- Behavior
- Mathematical Model
- Earthquake Direction
- Longitudinal Analysis
- **Transverse Analysis**



# Single-Mode Spectral Method / 4.4 (1-A)

**Concept: Structure Responds in Single Vibration Mode**



**Shape  $\equiv$  Deflection from Uniform Lateral Load**

# Single-Mode Spectral Method Steps

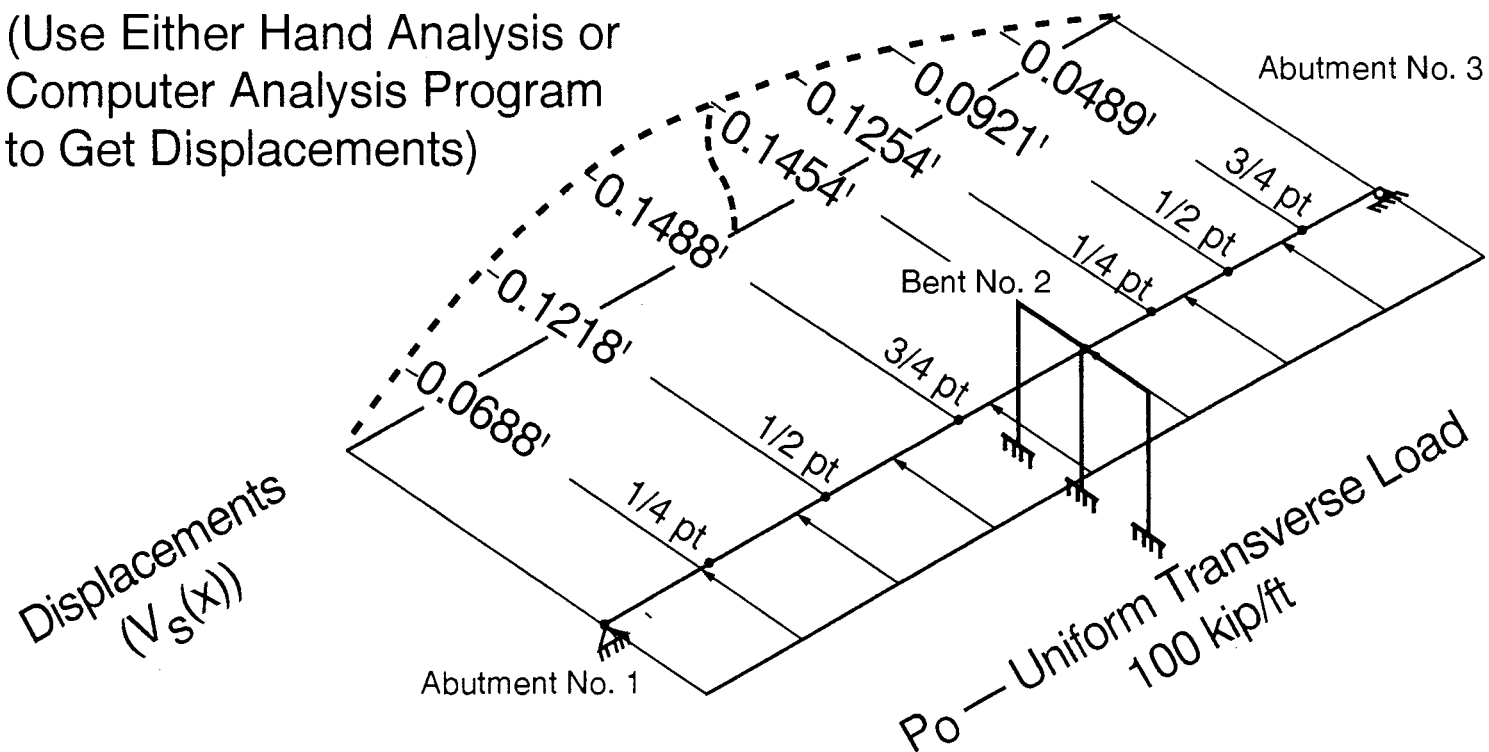
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1. Apply Uniform Load,  $P_0$  / Obtain Deflection,  $V_S(x)$
2. Calculate Modal Weighting Factors  $\left[ \begin{array}{l} \alpha = \int_0^L V_S(x) dx \\ \beta = \int_0^L w_S(x) V_S(x) dx \\ \gamma = \int_0^L w_S(x) V_S^2(x) dx \end{array} \right.$
3. Calculated Period,  $T = 2\pi \sqrt{\frac{\gamma}{P_0 g \alpha}}$
4. Calculate Inertial Loading,  
$$P_e(x) = \frac{\beta C_s}{\gamma} w(x) V_S(x)$$
5. Apply  $P_e(x)$  / Find Forces, Displacements

# Single-Mode Spectral Method / Step 1

- **Apply Uniform Load/Obtain Displacements**

(Use Either Hand Analysis or Computer Analysis Program to Get Displacements)

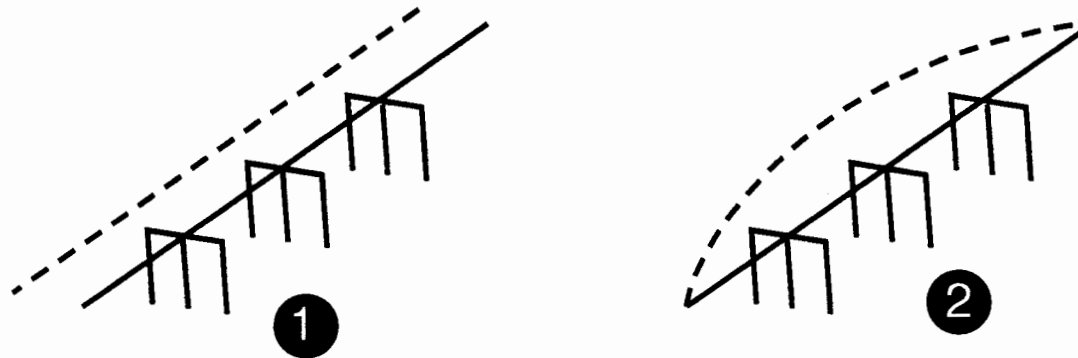


# 'Weighting Factors' $\alpha$ , $\beta$ , $\gamma$ / Step 2

---

## Transverse Seismic Movement

(Hypothetical 4-Span Example)

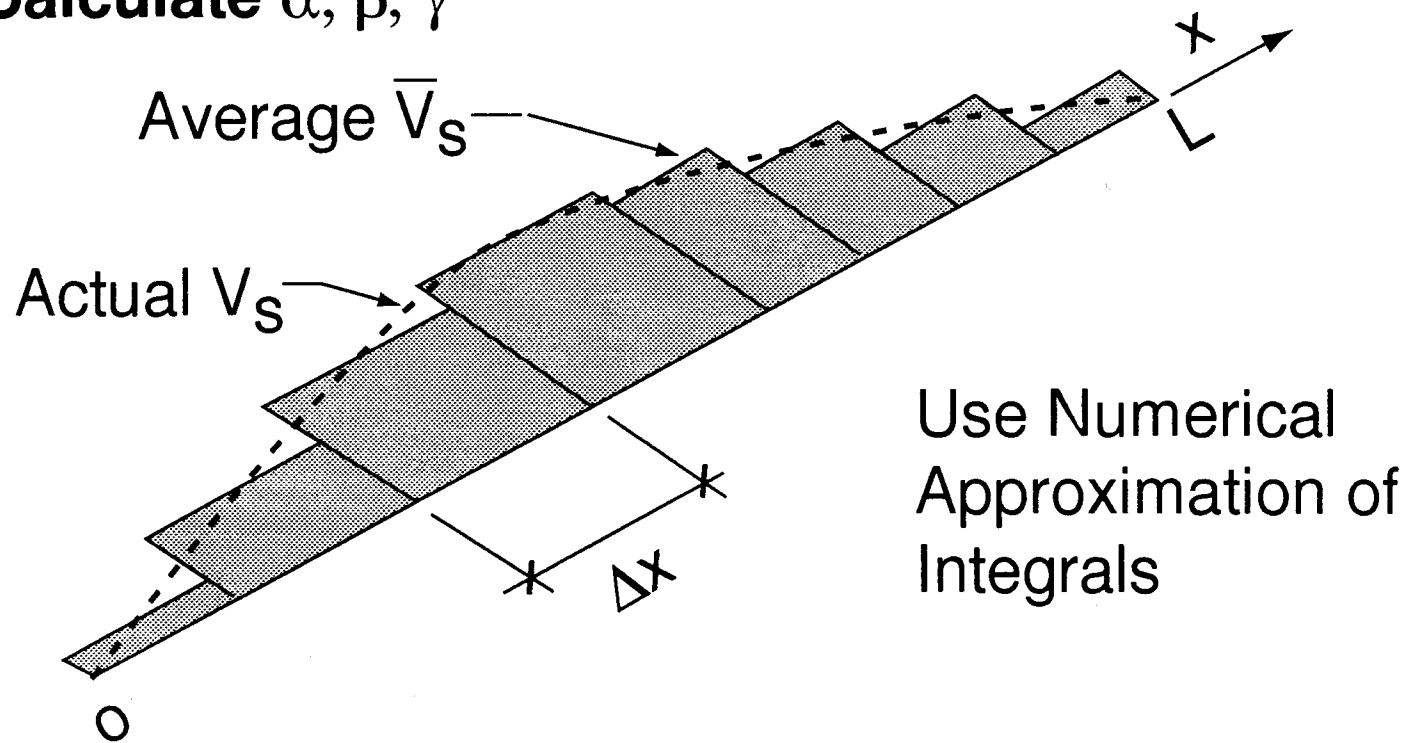


Weighting Factors Account for

- Resisting Elements (Piers, etc.) Deflect Differently
- Inertial Forces Vary in Accordance with Deflection

## Single-Mode Spectral Method / Step 2

- Calculate  $\alpha, \beta, \gamma$



# Single-Mode Spectral Method / Step 2 (continued)

$$\alpha = \int_0^L V_s dx \longrightarrow \alpha = \Sigma \bar{V}_s \Delta x \quad \text{AASHTO eqn. (4-5)}$$

$\alpha$  = Area Under Displacement Curve



$$\beta = \int_0^L w V_s dx \longrightarrow \beta = \Sigma w \bar{V}_s \Delta x \quad \text{(4-6)}$$

$\beta$  = Area Under Weight • Displacement



$$\gamma = \int_0^L w V_s^2 dx \longrightarrow \gamma = \Sigma w \bar{V}_s^2 \Delta x \quad \text{(4-7)}$$

$\gamma$  = Area Under Weight • Displacement<sup>2</sup>



# Single-Mode Spectral Method / Step 2 (continued)

Assumptions: $P_0 = 100.0 \text{ k/ft}$ $A = 0.28$ $g = 32.2 \text{ ft/sec}^2$ $2.5 \cdot A = 0.70$ $w(x) = 20.1 \text{ k/ft}$ $S = 1.2$							
1	2	3	4	5	6	7	8
Location	Node Distance $x$ (ft)	Tributary Length $dx$ (ft)	Displ Due to Uniform Loading $v_s(x)$ (ft)	$\alpha(x)$ (ft <sup>2</sup> )	$\beta(x)$ (k-ft)	$\gamma(x)$ (k-ft <sup>2</sup> )	Equiv. Static EQ Loading $P_e(x)$ (k-ft)
Abut No. 1	0.0	0.0	0.0000	1.22	24.55	1.69	0.00
1/4 pt	35.5	35.5	0.0688	3.38	68.02	6.98	8.03
1/2 pt	71.0	35.5	0.1218	4.80	96.54	13.19	14.23
3/4 pt	106.5	35.5	0.1488	5.22	104.94	15.43	17.37
Bent No. 2	142.0	35.5	0.1454	3.38	67.99	9.25	16.97
1/4 pt	167.0	25.0	0.1252	2.72	54.61	6.07	14.62
1/2 pt	192.0	25.0	0.0921	1.76	35.44	2.73	10.76
3/4 pt	217.0	25.0	0.0489	0.61	12.29	0.60	5.71
Abut No. 3	242.0	25.0	0.0000				0.00
Sum =		242.0		23.10	464.38	55.96	

**Weight**  
Include Structure, Barriers  
Overlay, Diaphragms, etc.

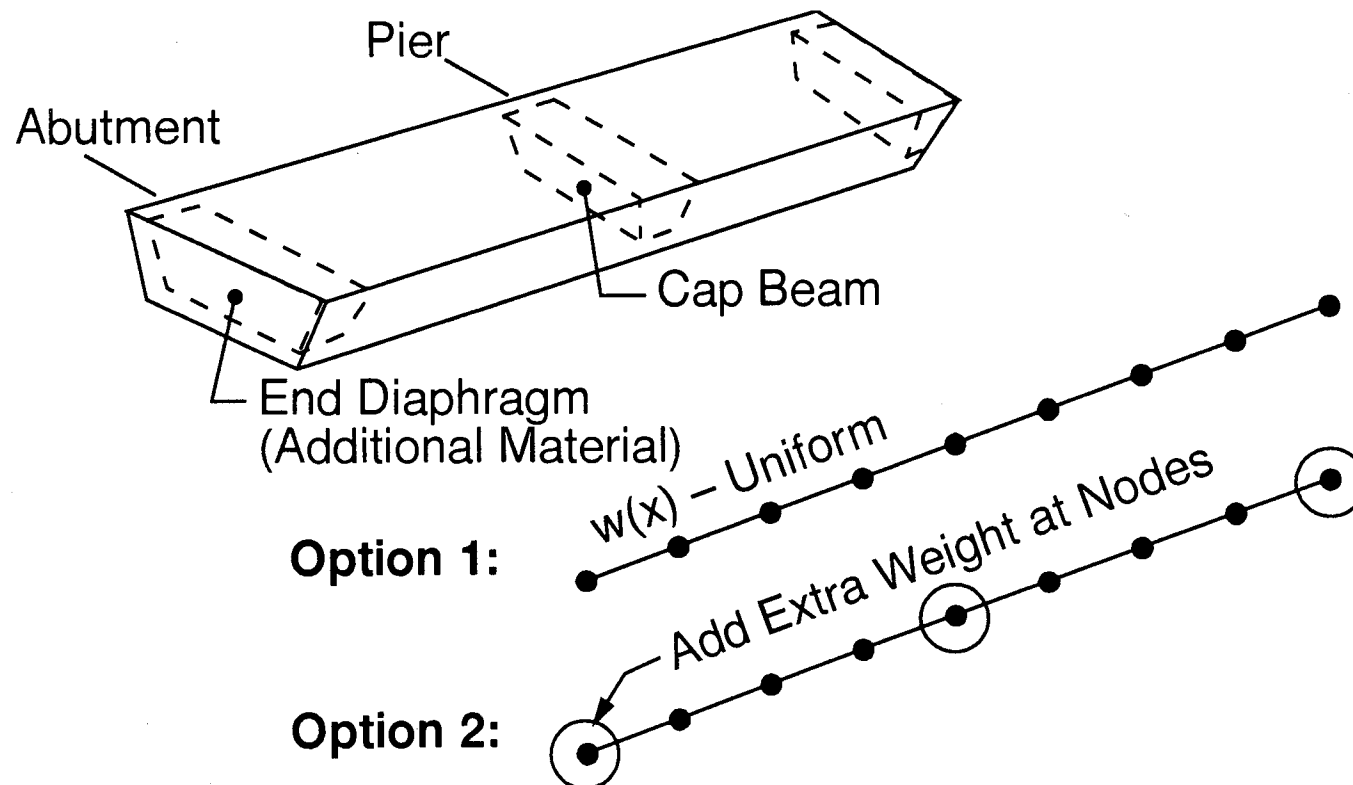
(Either Lump or  
Spread Evenly)

$$\alpha = 23.10 \text{ ft}^2$$

$$\beta = 464.4 \text{ kip ft}$$

$$\gamma = 55.96 \text{ k ft}^2$$

## Weight Distribution / Step 2





# Single-Mode Spectral Method / Step 3

---

- Calculate Period T

$$T = 2\pi \sqrt{\frac{\gamma}{P_o g \alpha}} = 2\pi \sqrt{\frac{55.96}{100 (32.2) 23.10}} \quad \text{Eqn (4-8)}$$

$$T = 0.17 \text{ sec}$$

Units:

$$T = 2\pi \sqrt{\frac{\text{kip ft}^2}{\left(\frac{\text{kip}}{\text{ft}}\right) \left(\frac{\text{ft}}{\text{sec}^2}\right) (\text{ft}^2)}} = \sqrt{\text{sec}^2} = \text{sec}$$

# Single-Mode Spectral Method / Step 4

---

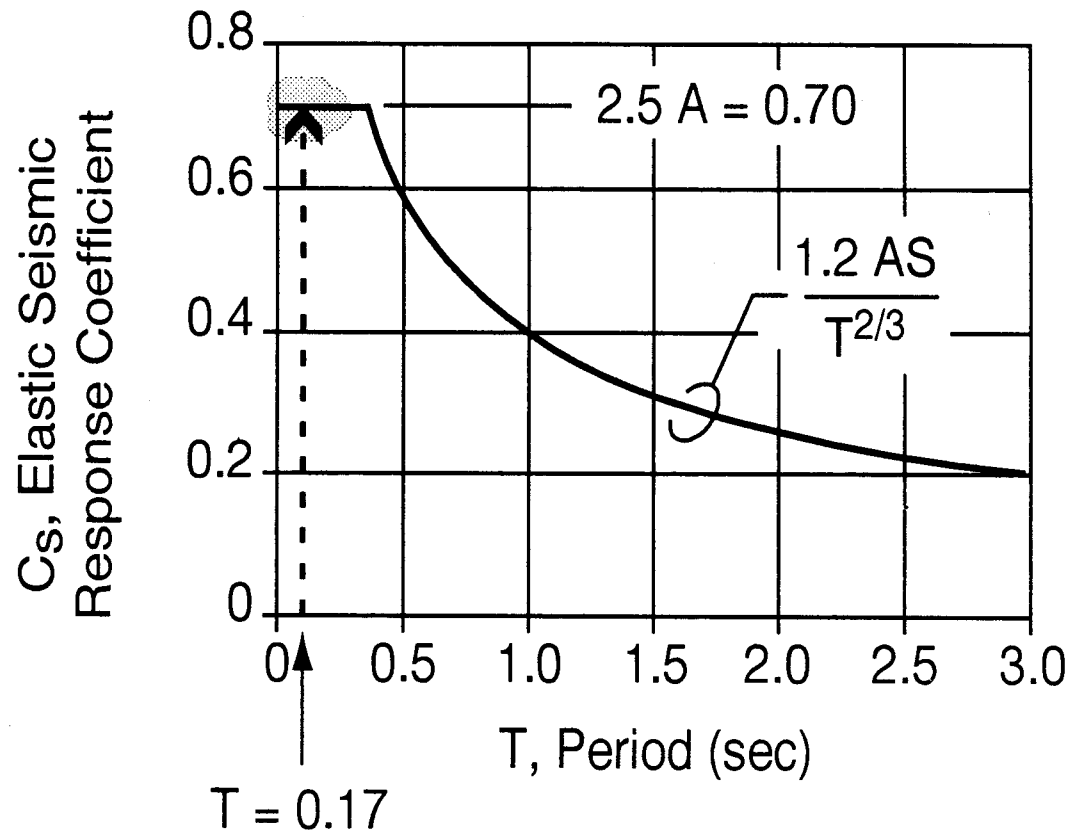
- Calculate Equivalent Static Earthquake Loading

Elastic Seismic  
Response  
Coefficient,  $C_S$

$$\left[ \begin{array}{l} C_S = \frac{1.2AS}{T^{2/3}} \leq 2.5 A \\ C_S = \frac{1.2(0.28)1.2}{(0.17)^{2/3}} \leq 2.5 (0.28) \\ C_S = 1.30 \leq 0.70 \end{array} \right.$$

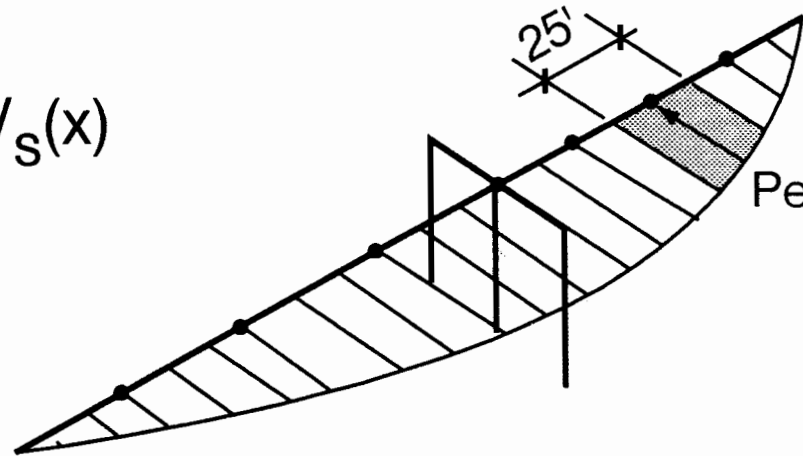
Controls

## Single-Mode Method / Step 4 (continued)



## Single-Mode Method / Step 4 (continued)

$$P_e(x) = \frac{\beta C_s}{\gamma} w(x) V_s(x)$$

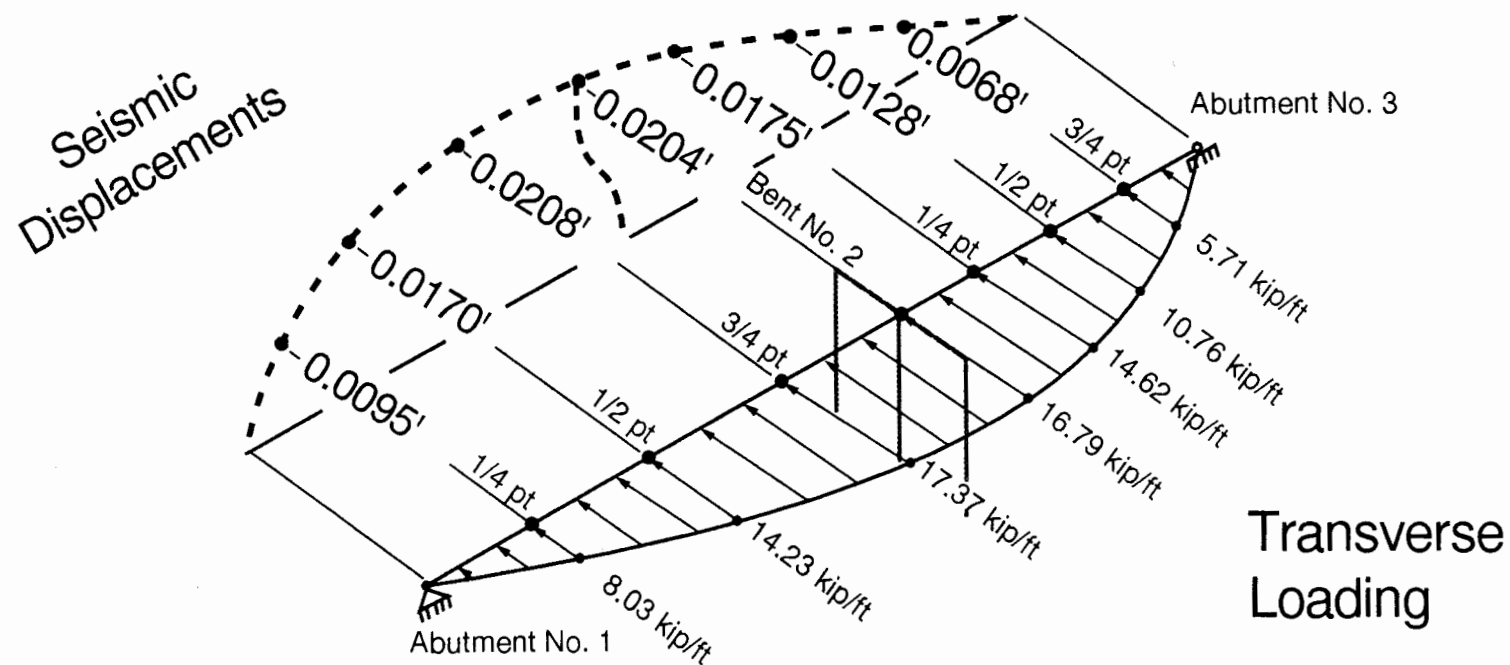


$$P_e = \frac{464.4}{55.96} (0.70) 20.1 (0.0921) = 10.76 \text{ kip/ft (Load Intensity)}$$

$$P_e = 10.76 (25) = 269 \text{ kip (Concentrated Load at Node)}$$

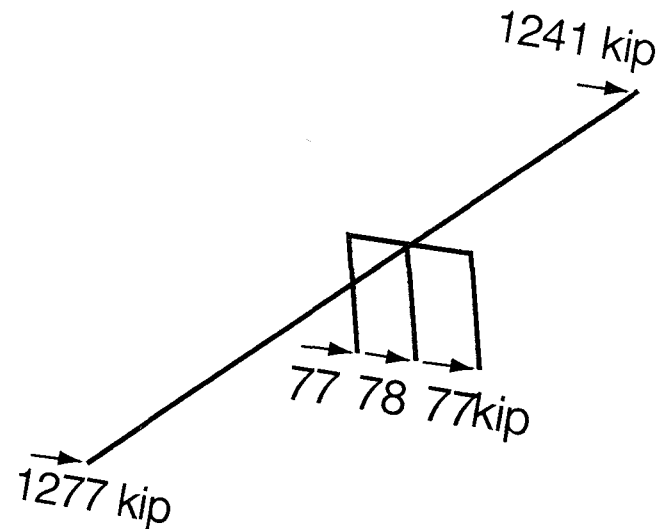
# Single-Mode Spectral Method / Step 5

- Apply  $P_e(x)$  / Find Forces, Displacements



# Transverse Loading Results

			Forces and Moments				
			Longitudinal		Transverse		Axial (kips)
			Shear (kips)	Moment (kip-ft)	Shear (kips)	Moment (kip-ft)	
Abutment No. 1			0	0	1277	583	0
Bent No. 2	Center	Top	0	0	77.8	1062	0
		Bottom	0	0	77.8	910	0
	Outboard	Top	8.1	110	77.2	1053	42.5
		Bottom	8.1	94.7	77.2	902	42.5
Abutment No. 3			0	0	1241	828	0





# Session 4

## Example Design of Two-Span Bridge

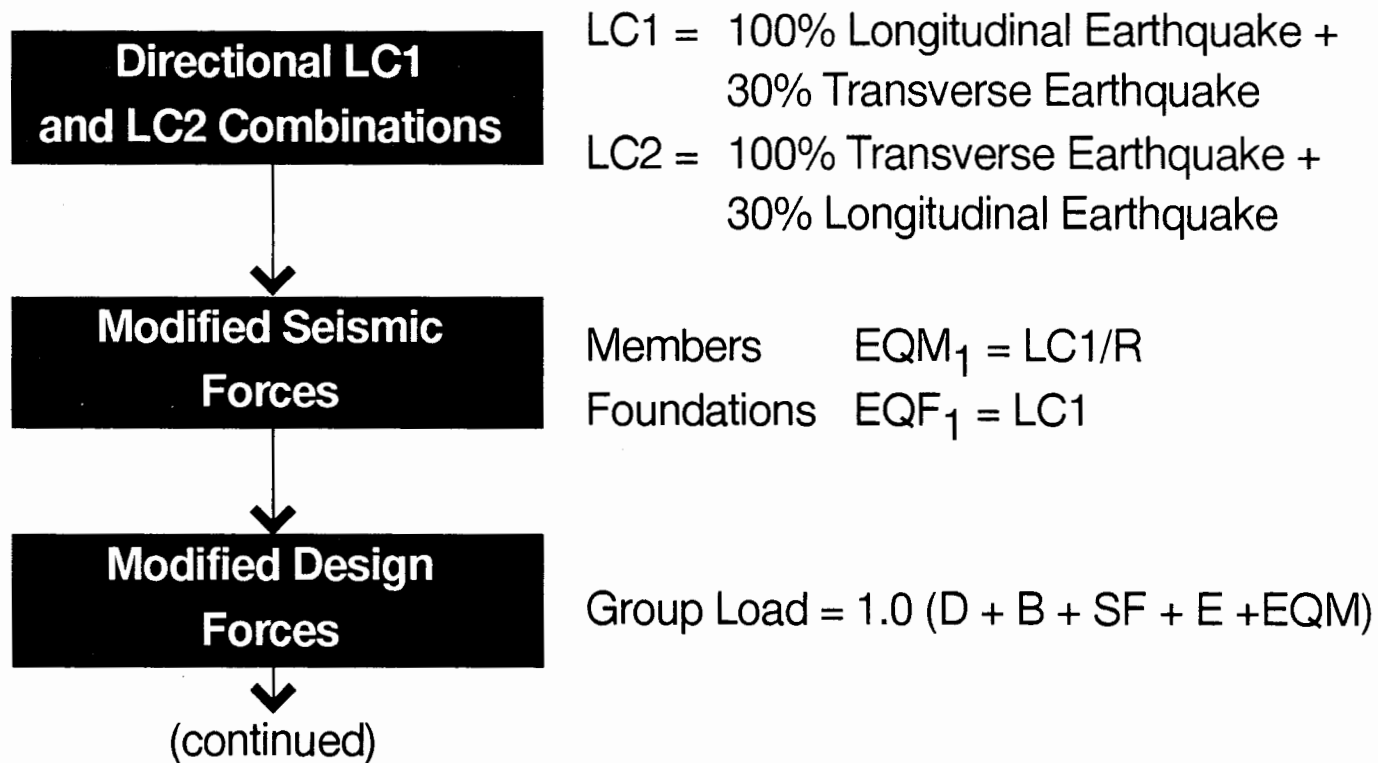
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- **Elastic Forces** —→ **Design Forces**  
(Including Column Flexural Design)
- **Design Columns**
- **Design Column Footings**
- **Abutment Issues**



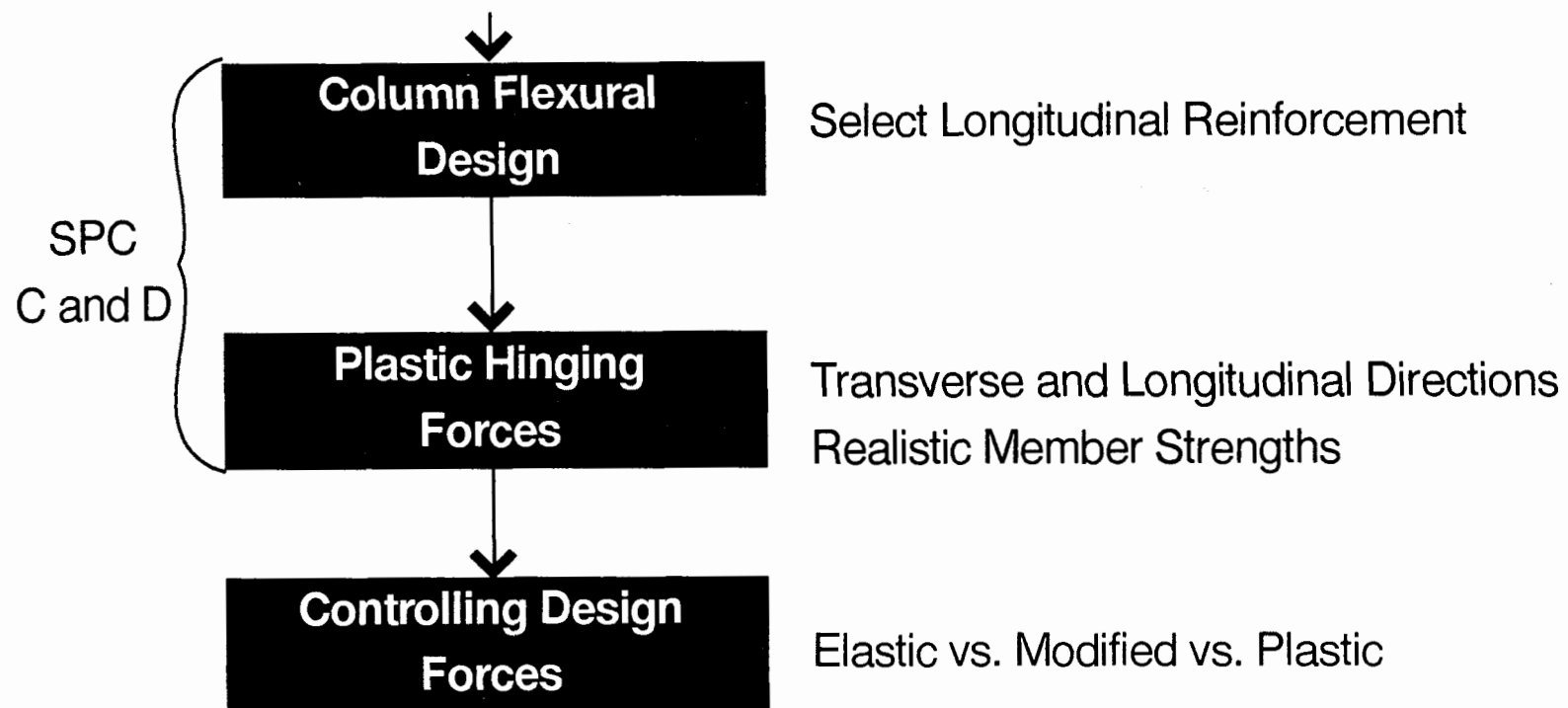
# From Elastic Seismic Forces To Design Forces

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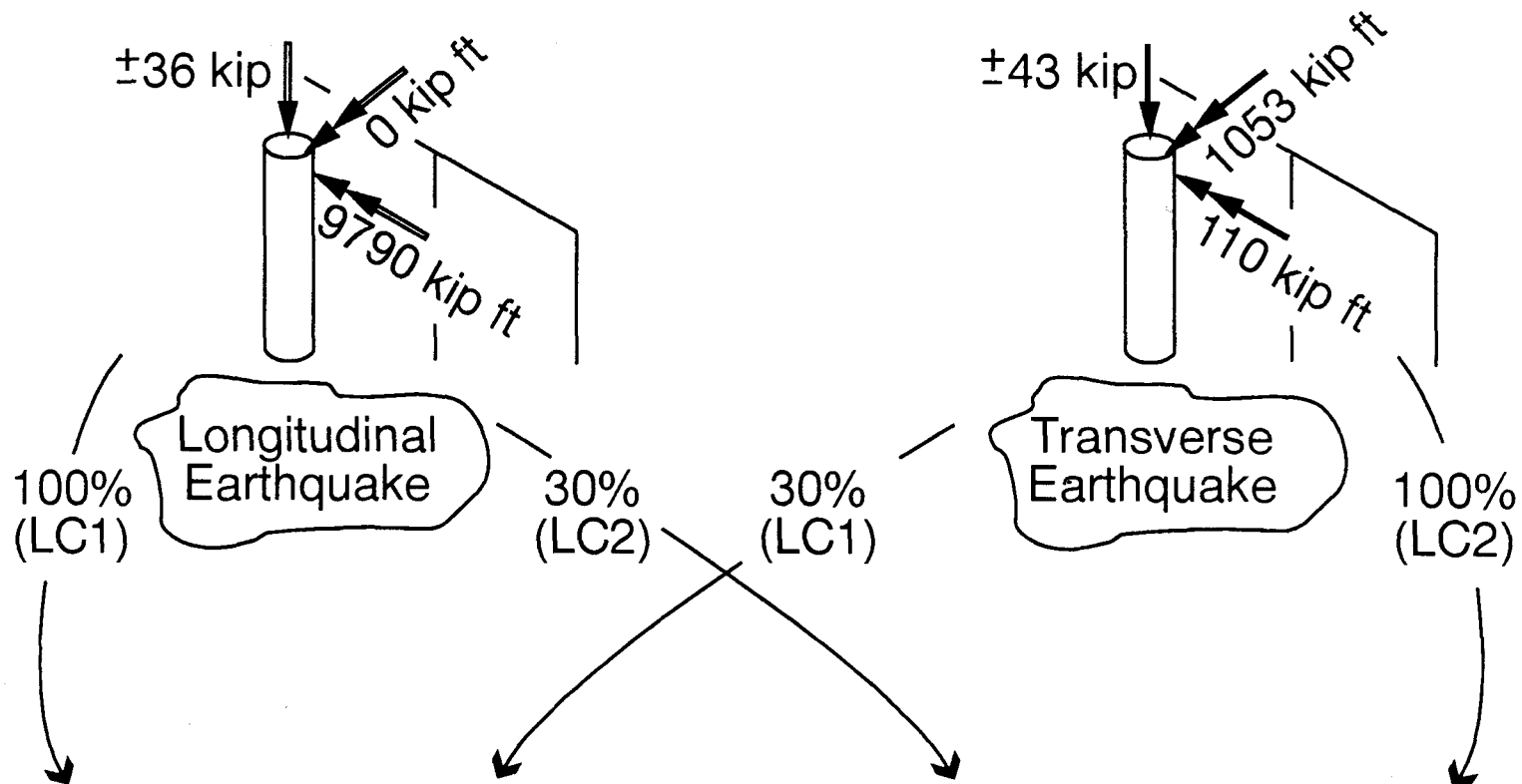


# From Elastic Seismic Forces To Design Forces

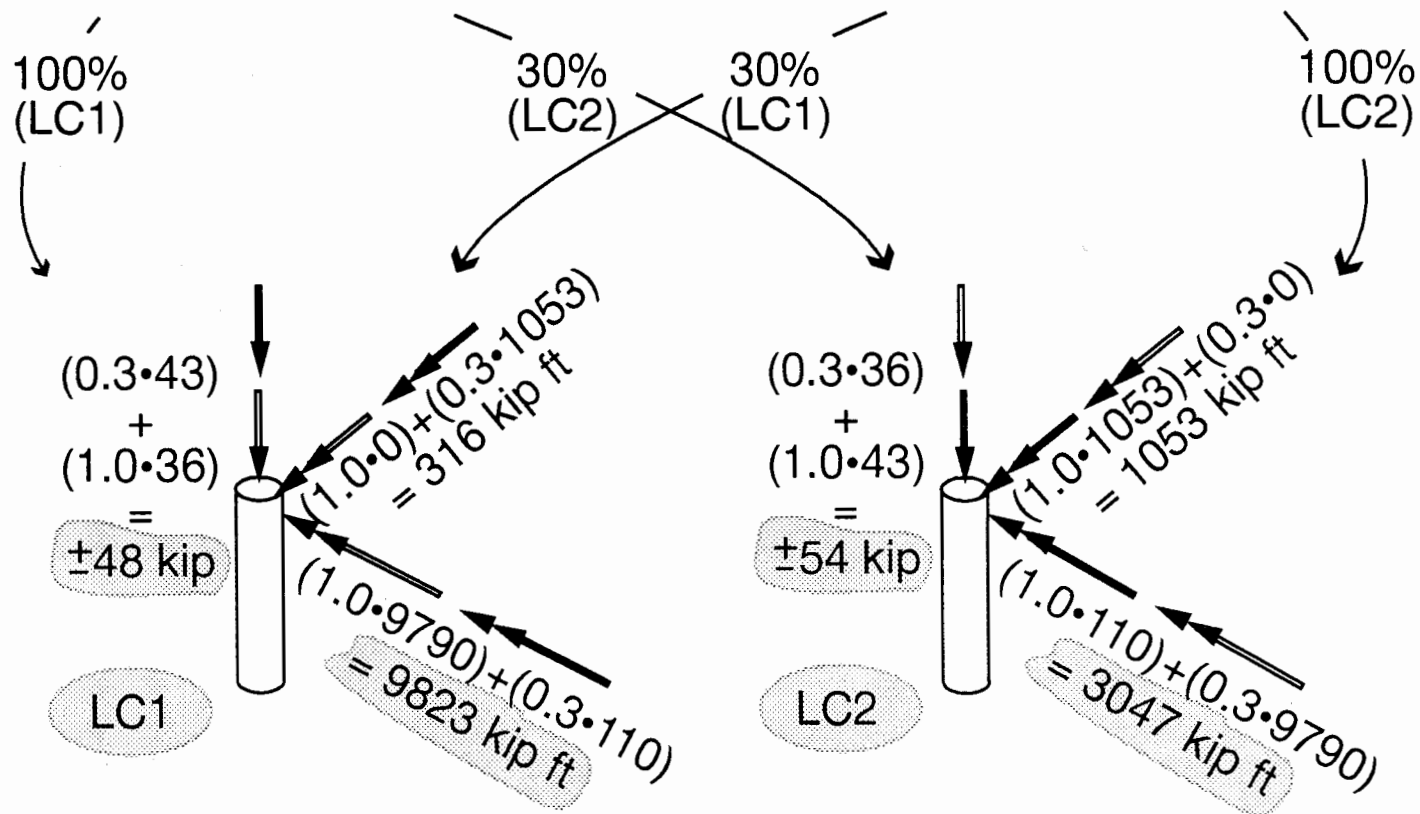
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# Orthogonal Seismic Force Combination



# Orthogonal Seismic Force Combination



# Determine Modified Seismic Forces

**Exterior  
Column**

**Top**

$$EQM_1 = \frac{LC1}{R}$$

$R = 5$  Multiple Column  
Bent Moments

$$EQM_1 = \begin{cases} M_T = 316/5 = 63 \text{ kip ft} \\ M_L = 9823/5 = 1965 \text{ kip ft} \\ P = \pm 48/1 = \pm 48 \text{ kip} \end{cases}$$

$$EQM_2 = \begin{cases} M_T = 211 \text{ kip ft} \\ M_L = 609 \text{ kip ft} \\ P = \pm 54 \text{ kip} \end{cases}$$

(continued)

Use  $R = 1$  for Axial Load and Shear

## Determine Modified Seismic Forces (continued)

---

**Bottom**  
(Modified Forces for Foundation)

$EQF = \frac{LC1}{1.0}$

$R = 1$  Footing

SPC C and D

$EQF_1 = \begin{cases} M_T = 271 \text{ kip ft} \\ M_L = 9509 \text{ kip ft} \end{cases}$

$EQF_2 = \begin{cases} M_T = 902 \text{ kip ft} \\ M_L = 2939 \text{ kip ft} \end{cases}$

## Combine into Group Load to Get Modified Design Forces

---

Group Load = 1.0 (D + ~~B~~ + ~~SF~~ + ~~E~~ + EQM) ...for this Example

0      0      0

Substitute EQF  
for Foundations

Replaces Division I, Group VII

## Combine into Group Load to Get Modified Design Forces (continued)

---

$$\text{Group Load} = D + EQM$$

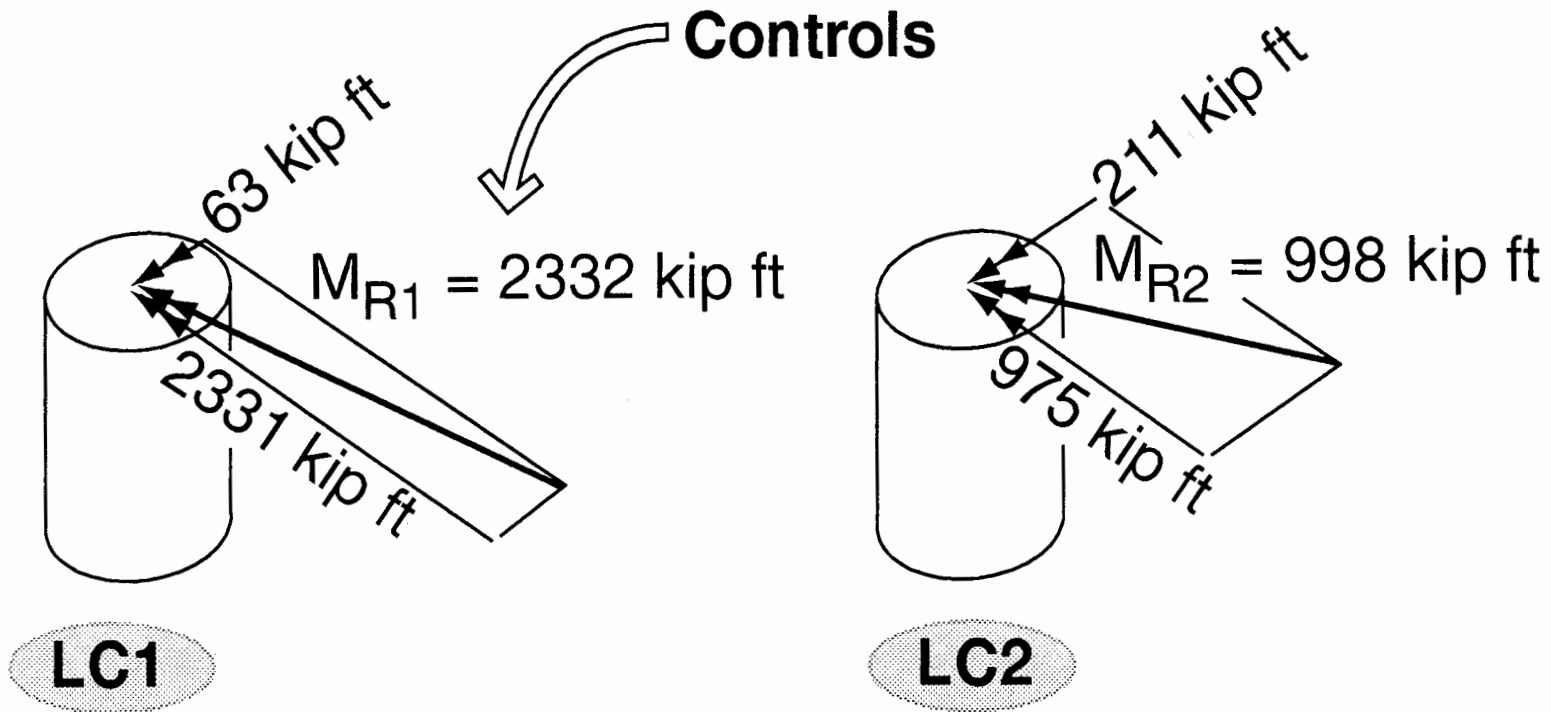
**Exterior  
Column  
Top**

$$\text{LC1} \left\{ \begin{array}{l} M_T = 63 \text{ kip ft} \\ M_L = 366 + 1965 = 2331 \text{ kip ft} \\ P_{\max} = 1098 + 48 = 1146 \text{ kip} \\ P_{\min} = 1098 - 48 = 1050 \text{ kip} \end{array} \right.$$

$$\text{LC2} \left\{ \begin{array}{l} M_T = 211 \text{ kip ft} \\ M_L = 975 \text{ kip ft} \\ P_{\max} = 1152 \text{ kip} \\ P_{\min} = 1044 \text{ kip} \end{array} \right.$$



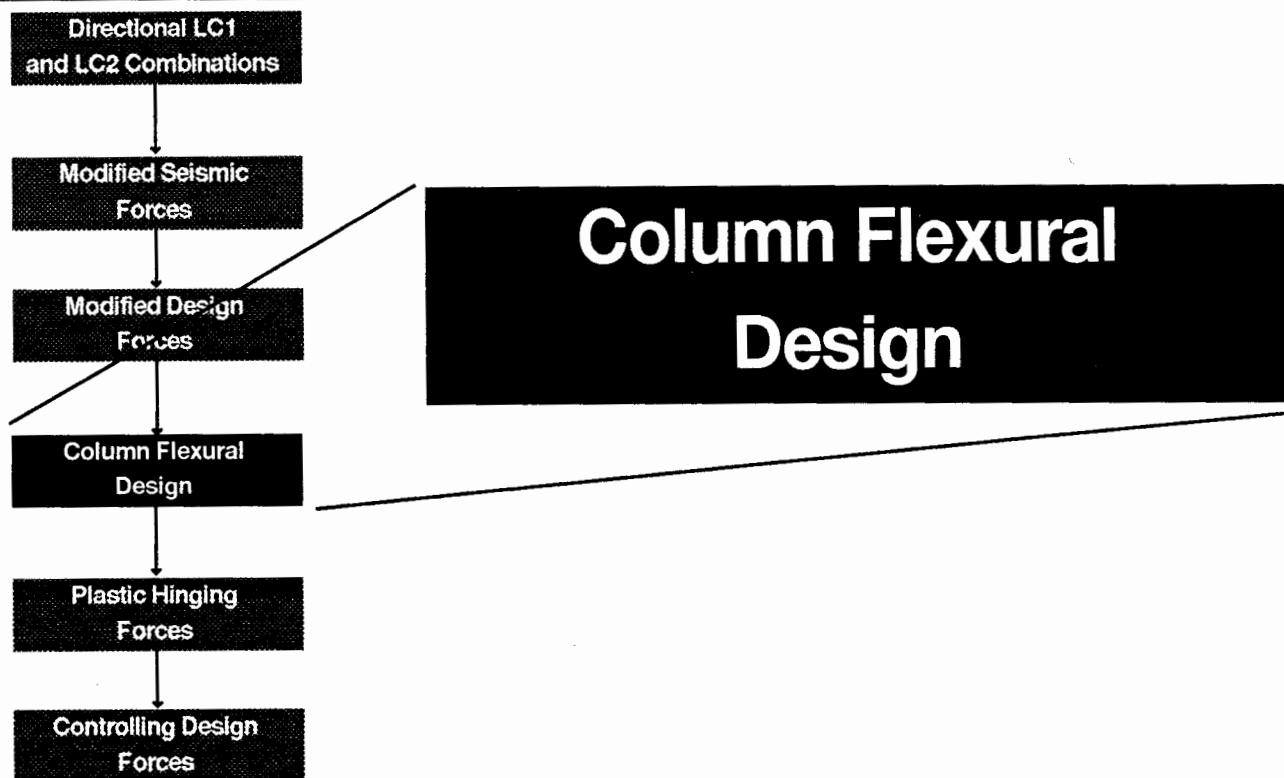
# Determine Controlling Moment for Flexural Design



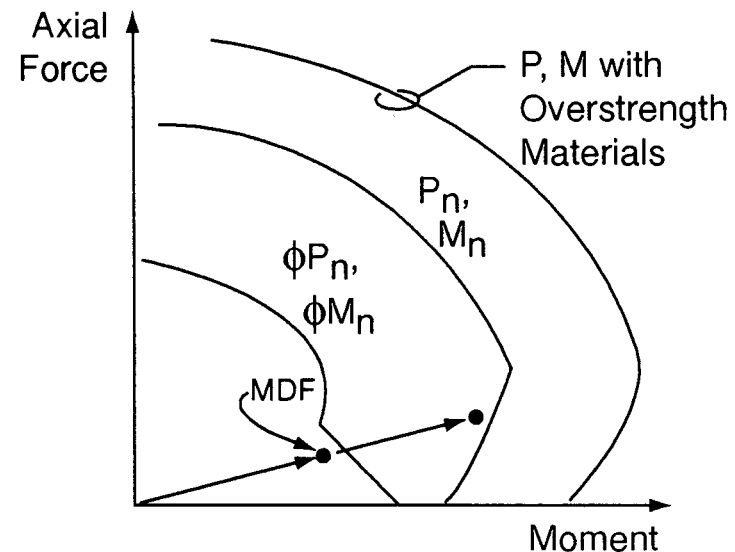
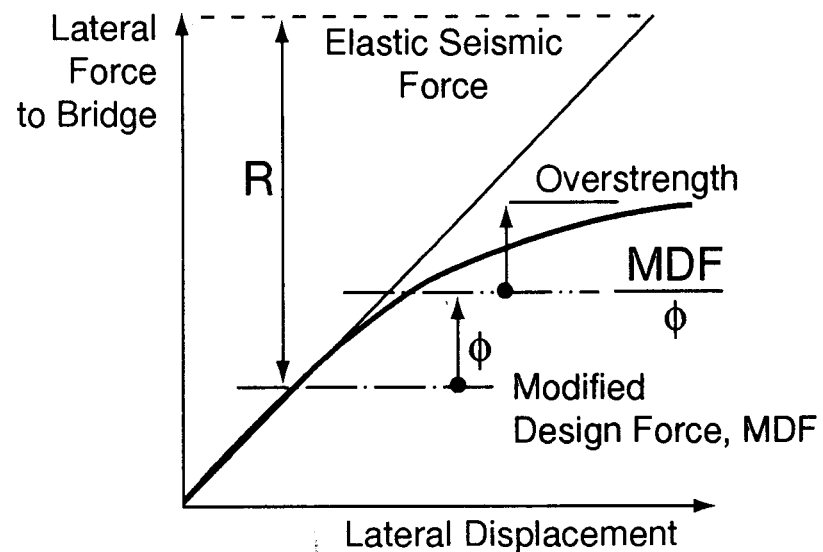
# Column Design Forces / Flexure

	Per AASHTO	Elastic R=1	Modified R=5
<b>Axial</b>	Use Full Elastic Value	1050 kip 1146 kip	Not Used
		LC1 Controls	
<b>Moment</b>	Use Modified Value	Not Used	2332 kip ft
		Longitudinal Load Case (Primary)	

# From Elastic Seismic Forces To Design Forces



# Realistic Forces and Internal Moments



## Controlling Forces for Design

---

- Design Shear Resistance, Connections, Foundations, etc., for 'Overstrength Forces' (Plastic Hinging Forces)

**or**

- Design for Elastic Forces

**Whichever Is Smaller**

# Handling Material “Overstrength”

---

- **Realistic Values**

$$f'_c \approx 1.5 \cdot (\text{nominal } f'_c) \dots$$

Confinement  
Conservative Mix Design  
Strength Gain with Time

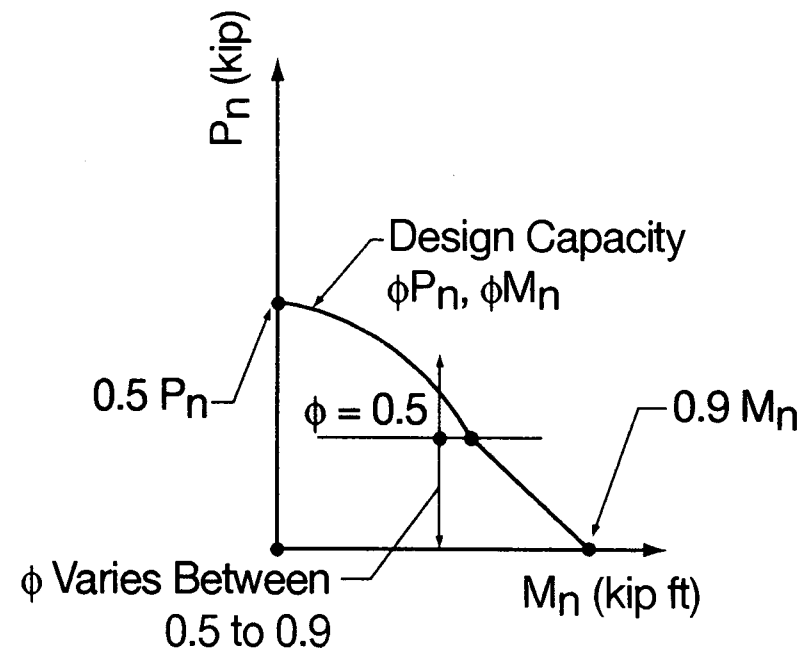
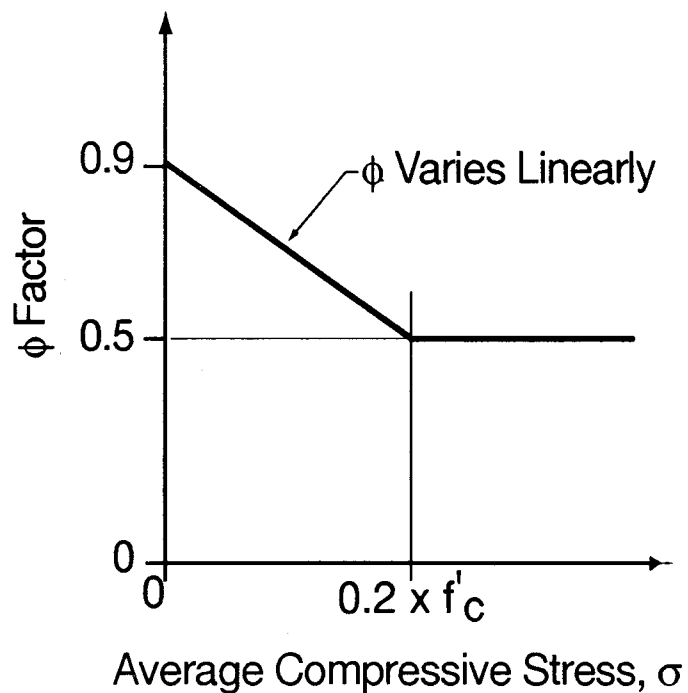
$$f_y \approx 1.25 \cdot (\text{nominal } f_y) \dots$$

95% or More Bars Have  
 $f_y$  Greater than Nominal  
Strain Hardening

- **AASHTO Allows Simple 1.3 Increase in  $\phi$  Factor**

$$M_{\text{actual}} = 1.3 M_n$$

# Strength Reduction Factor for SPC C and D



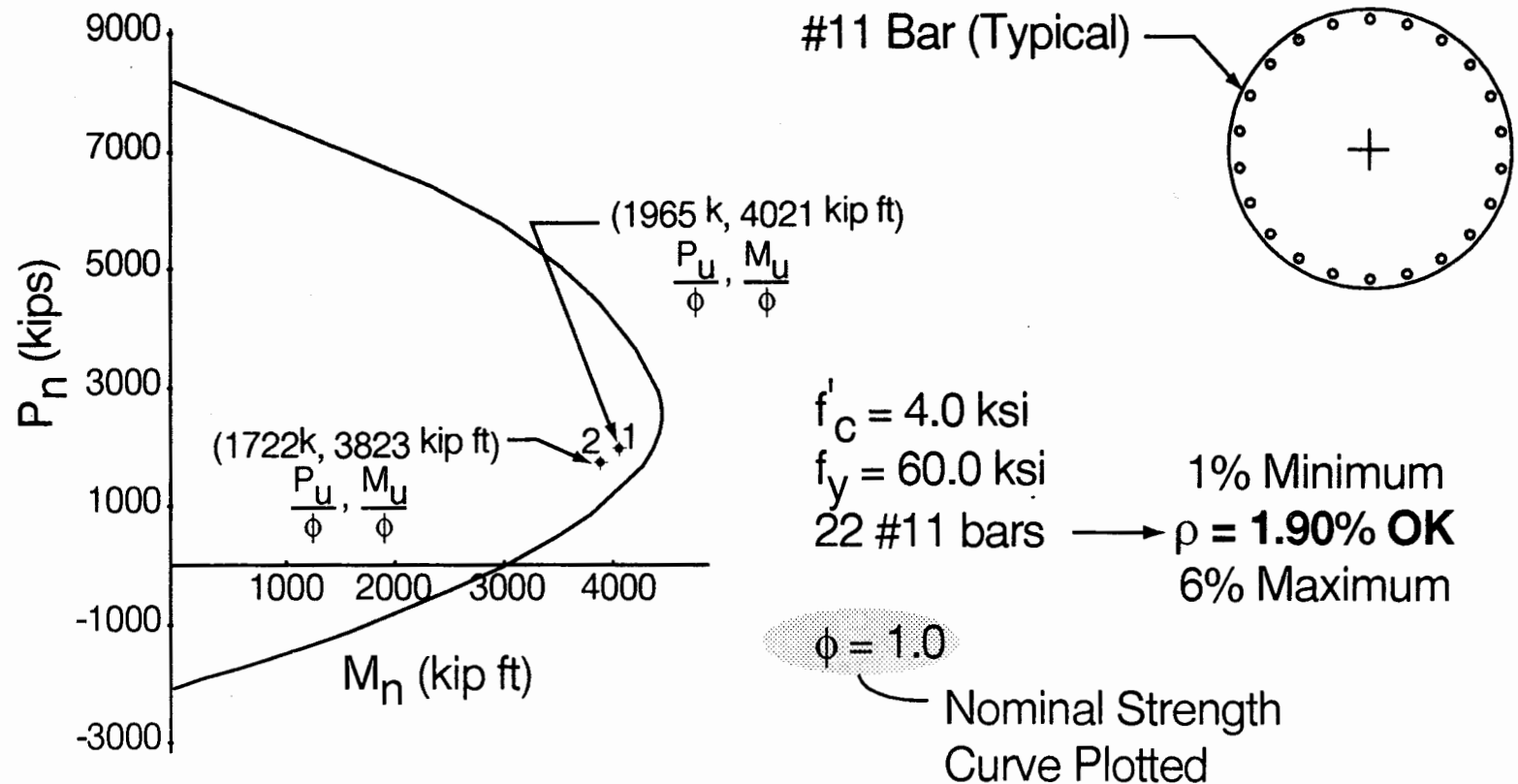
# Required Strength of Column

	$P$ (kips)	$\sigma$ (psi)	$0.2 f'_c$ (psi)	$\phi$	$\frac{P}{\phi}$ (kip)	$\frac{M}{\phi}$ (kip ft)
Max	1146	633	$\left. \begin{array}{l} 633 \\ 580 \end{array} \right\} < 800 \rightarrow \text{Interpolate}$ $\phi$	0.58	1965	4021
Min	1050	580		0.61	1722	3823

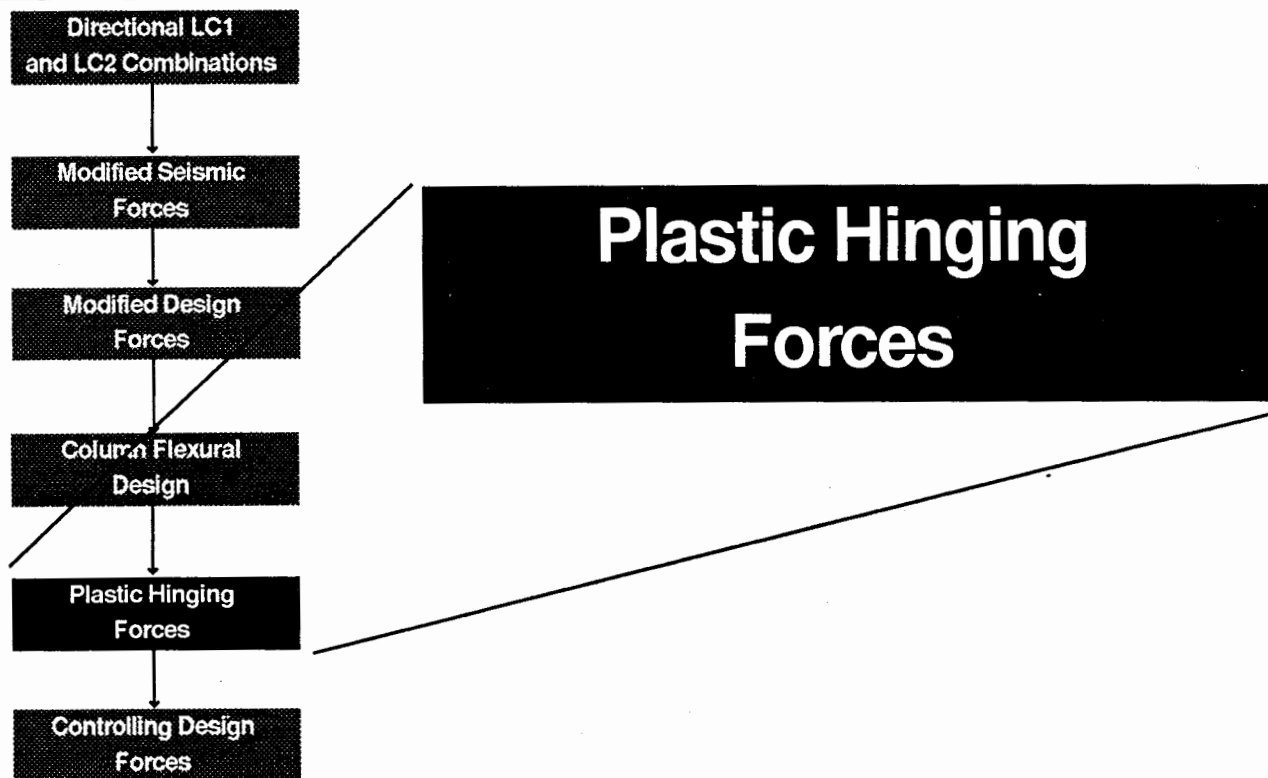
Column Diameter = 4 ft



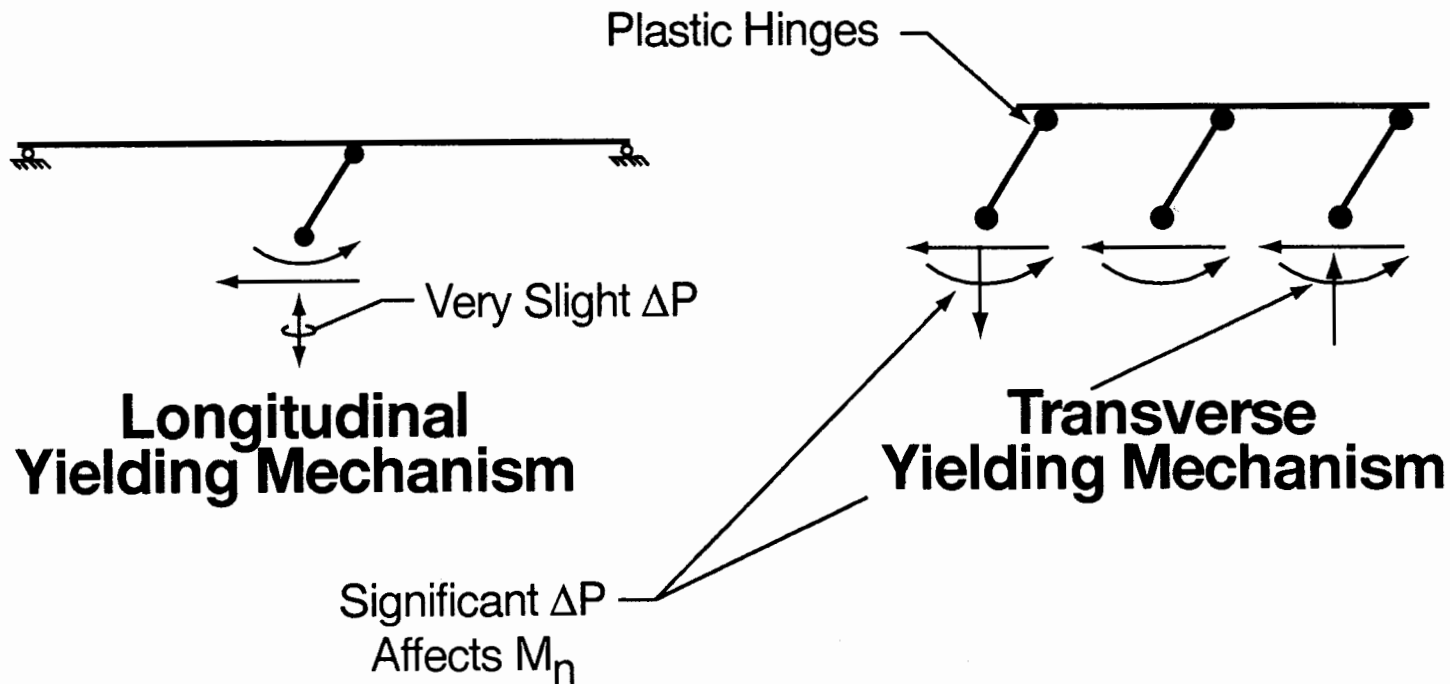
# Select Longitudinal Reinforcement



# From Elastic Seismic Forces to Design Forces



# Plastic Hinging Behavior



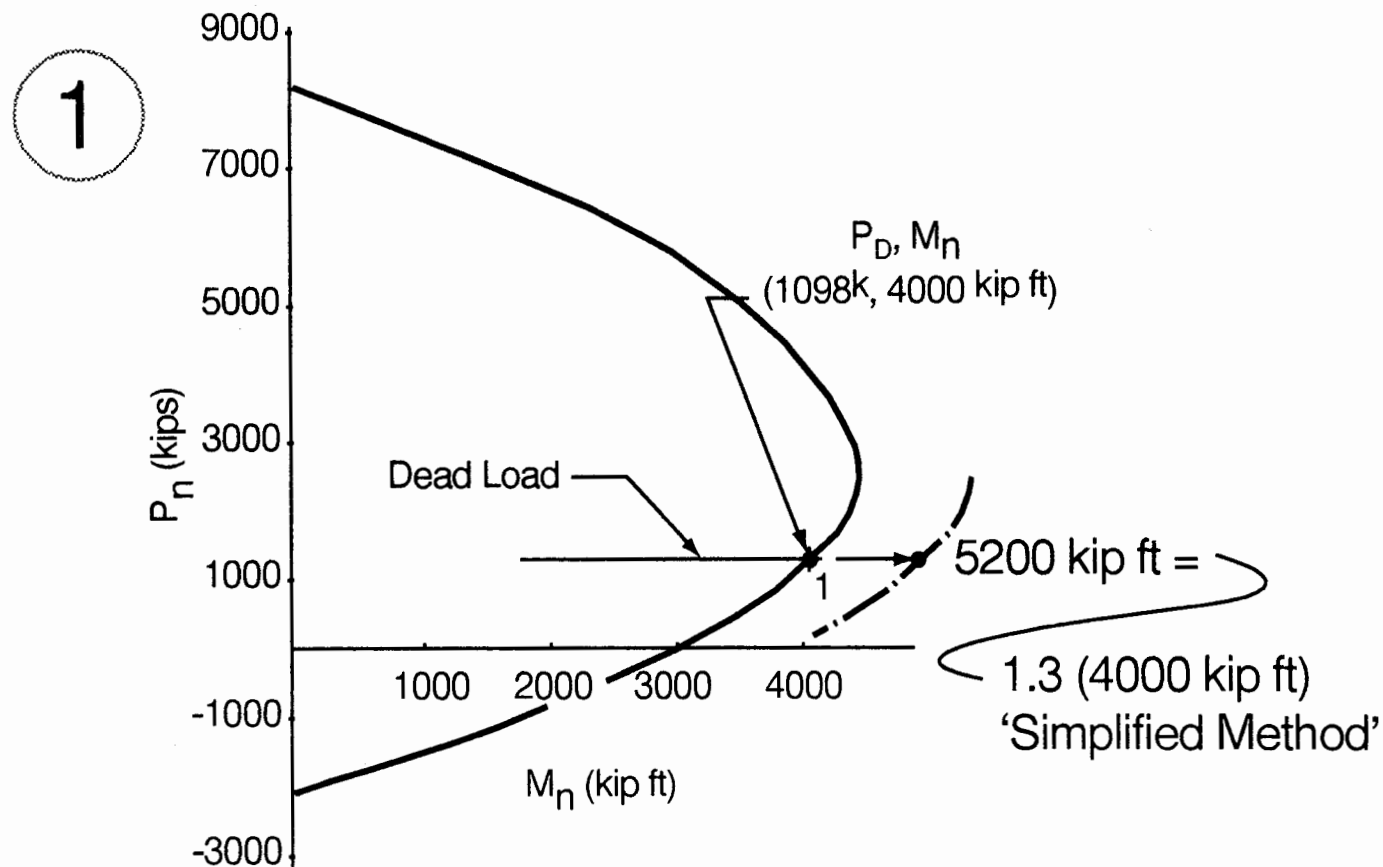
## Outline / Plastic Hinging Forces / Multiple Column Bents

---

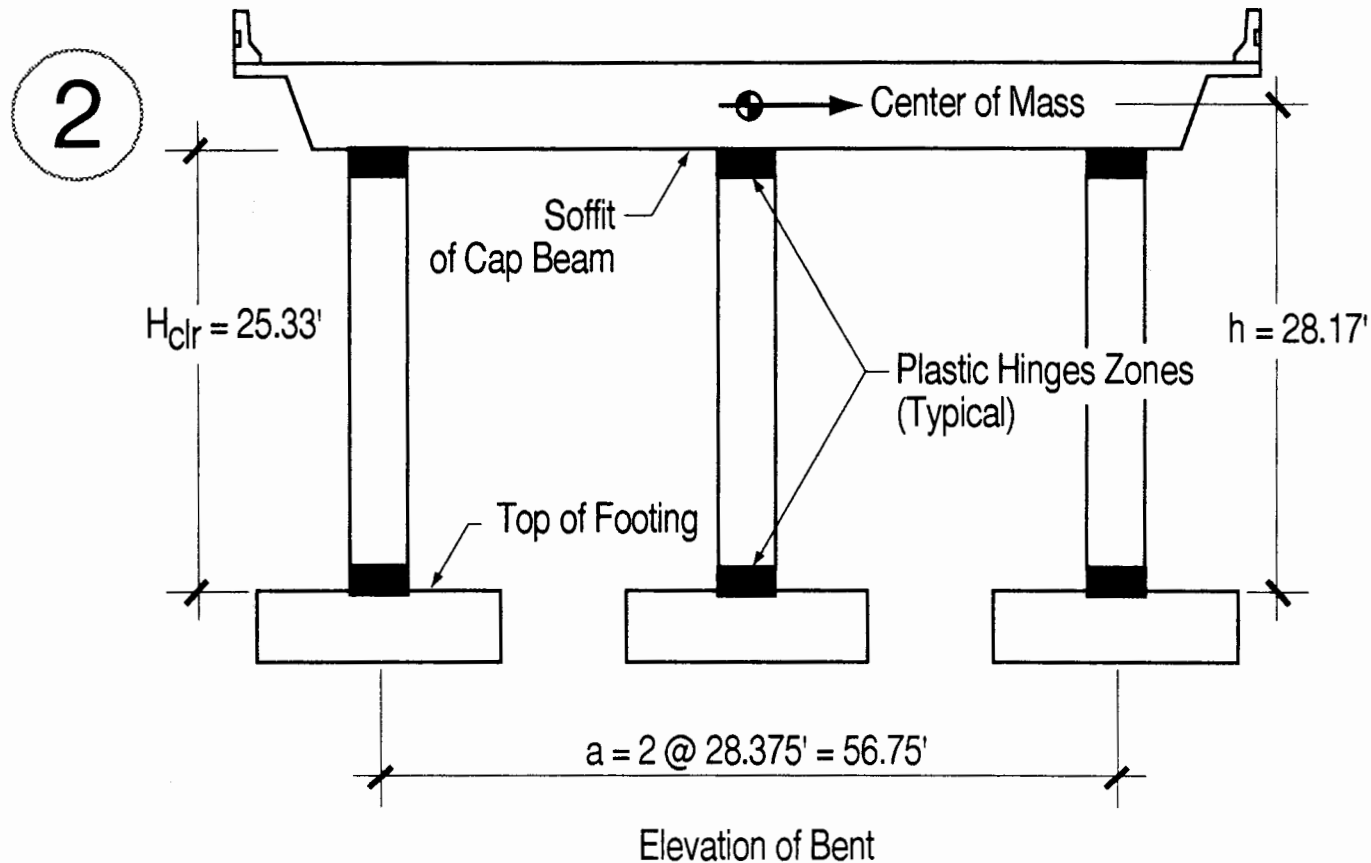
- 7.2.2 (B) (I-A)

1. Determine  $M_p$  for Axial Dead Load,  $P_D$
  2. Calculate Column Shears,  $V$
  - 3. Apply Total Shear,  $\Sigma V$ , to Bent and Find  $\Delta P$
  4. Determine Revised  $M_p$  and New Column Shears
- Repeat if Axial Force Has Not Converged

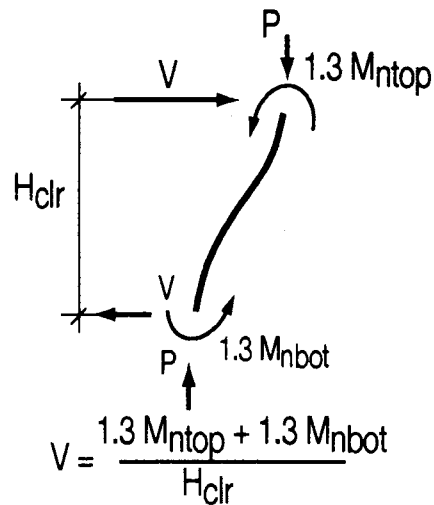
# Column Plastic Hinging Forces / Step 1



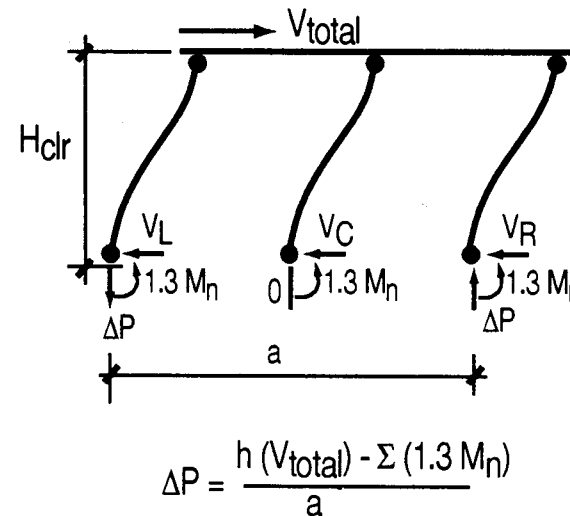
## Transverse Plastic Hinging / Step 2



# Transverse Plastic Hinging (continued)



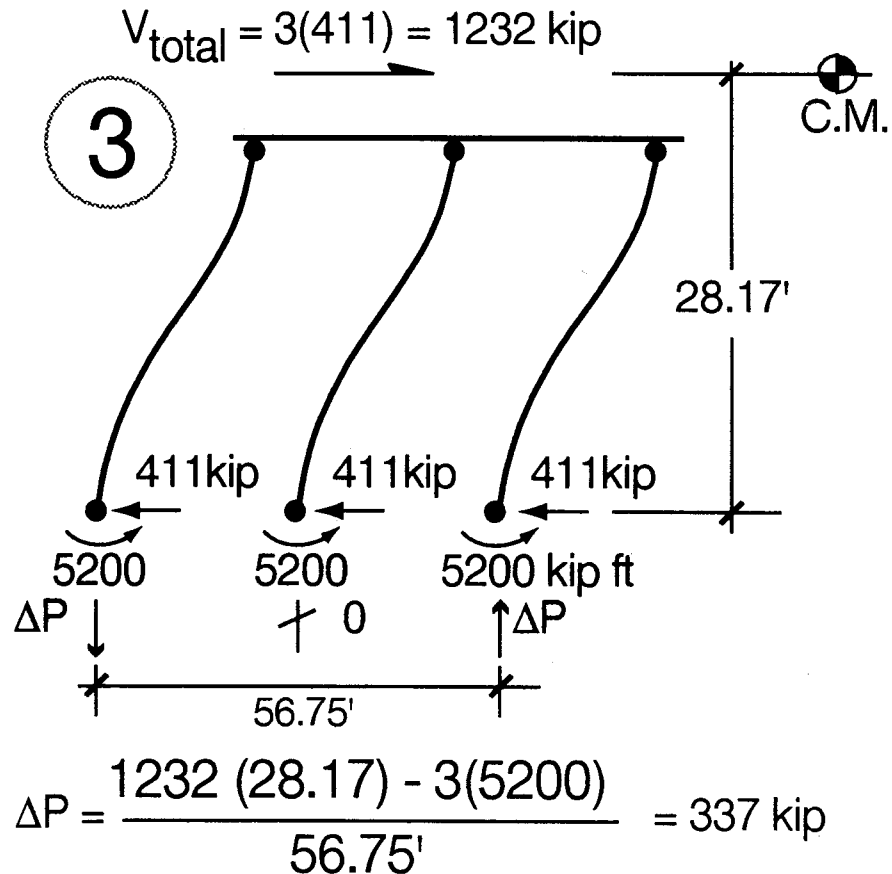
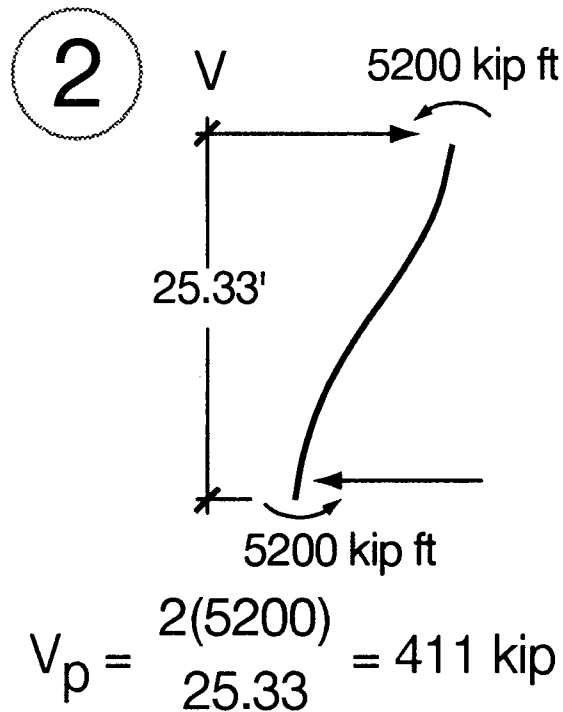
FBD of a Column  
with Plastic Hinges



Seismic Forces on Columns when  
Mechanism Has Formed

- Realistic  $V$  Depends on **Realistic**  $M_n$  ... Overstrength
- Realistic  $M_n$  Depends on **Realistic** Material Properties and  $\Delta P$

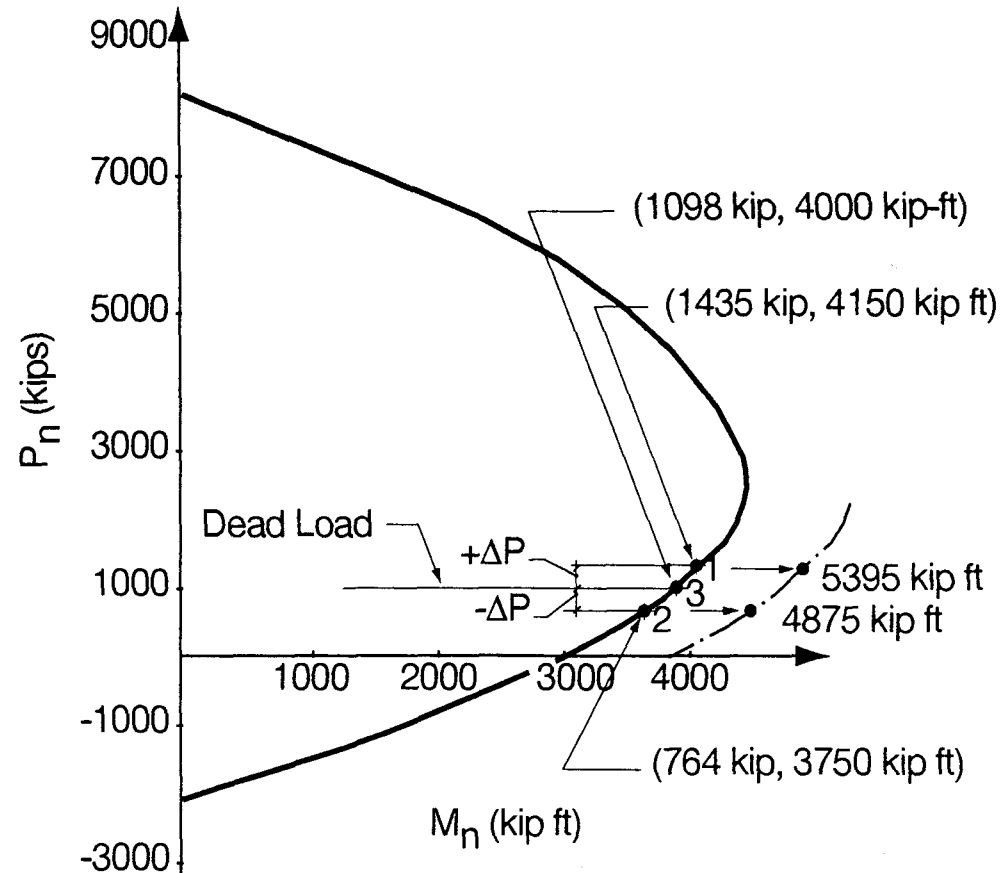
# Column Plastic Hinging Forces / Steps 2 and 3





# Column Plastic Hinging Forces / Step 4

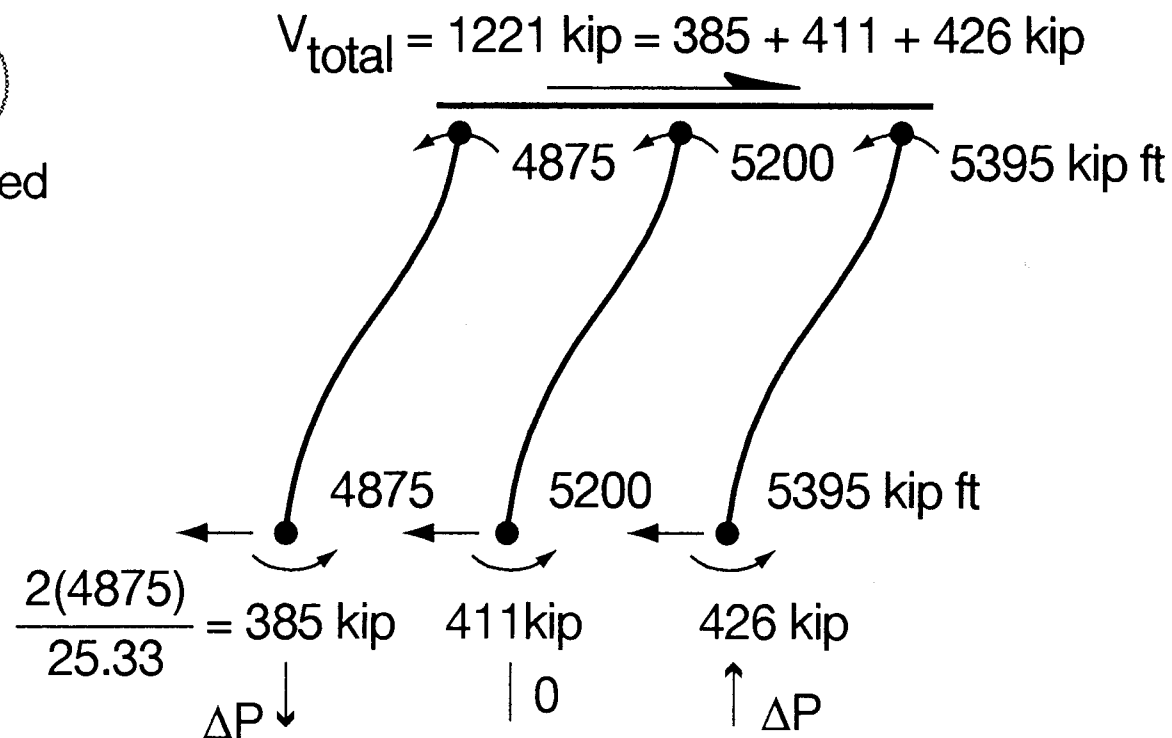
4



# Plastic Hinging / Second Cycle

4

Continued



$\Delta P = 334 \text{ kip}$  (vs. 337 kip Previous Value, Say OK)

# Summary Plastic Hinging Forces

---

## Transverse

Minimum Axial Load

$$P_p = 764 \text{ kip}$$

$$M_p = 4875 \text{ kip ft}$$

$$V_p = 385 \text{ kip}$$

Maximum Axial Load

$$P_p = 1432 \text{ kip}$$

$$M_p = 5395 \text{ kip ft}$$

$$V_p = 426 \text{ kip}$$

## Longitudinal

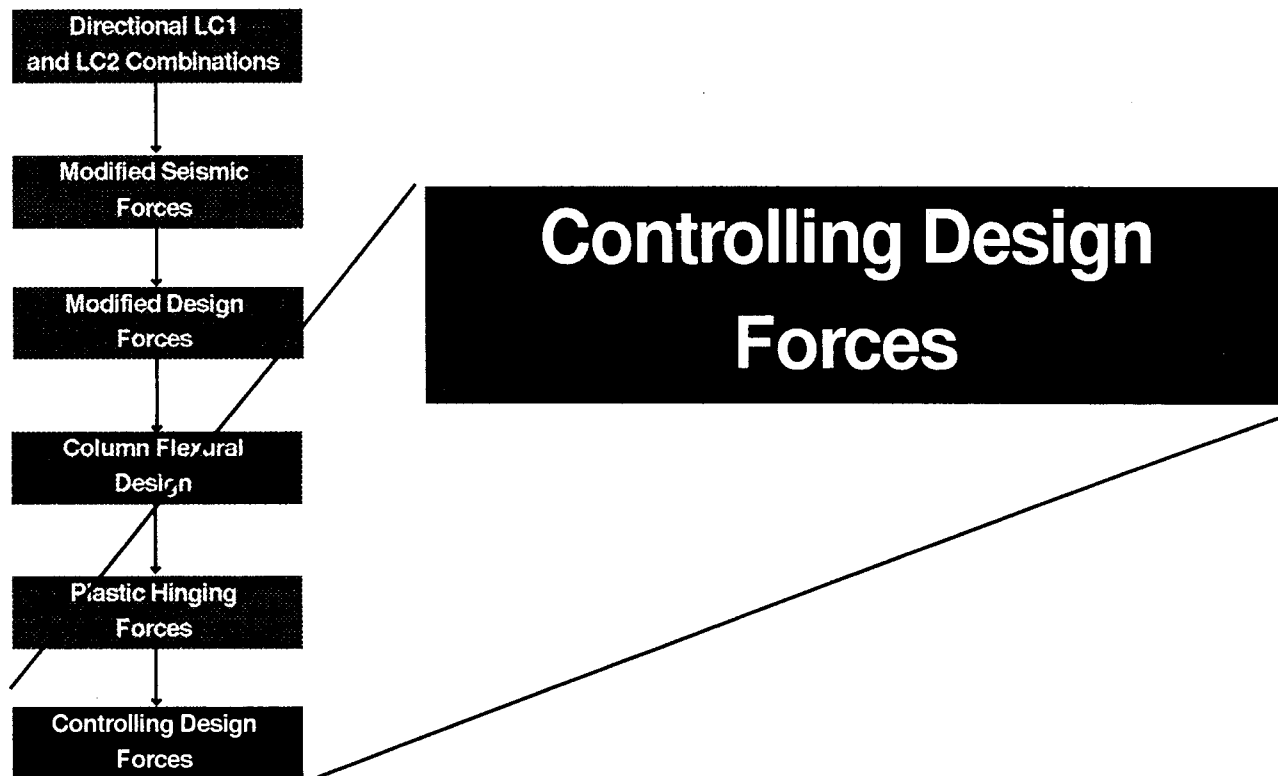
$$P_p \sim 1098 \text{ kip}$$

$$M_p \sim 5200 \text{ kip ft}$$

$$V_p \sim 411 \text{ kip}$$

Bracketed by  
Transverse Value

# From Elastic Seismic Forces to Design Forces



# Column Design / Shear

	Elastic/Modified (R=1/Group Load)		Plastic Hinging		
	LC1	LC2	Long.	Trans. Max.	Trans. Min.
<b>Shear</b>	762 kip	248 kip	411 kip	426 kip	385 kip
	(Resultants)				
<b>Axial*</b>	1146 kip	1152 kip	1098 kip	1432 kip	764 kip
	1050 kip	1044 kip			

Per AASHTO: Use Whichever Is Least

\*Note:  $V_c$  Depends on Axial Load

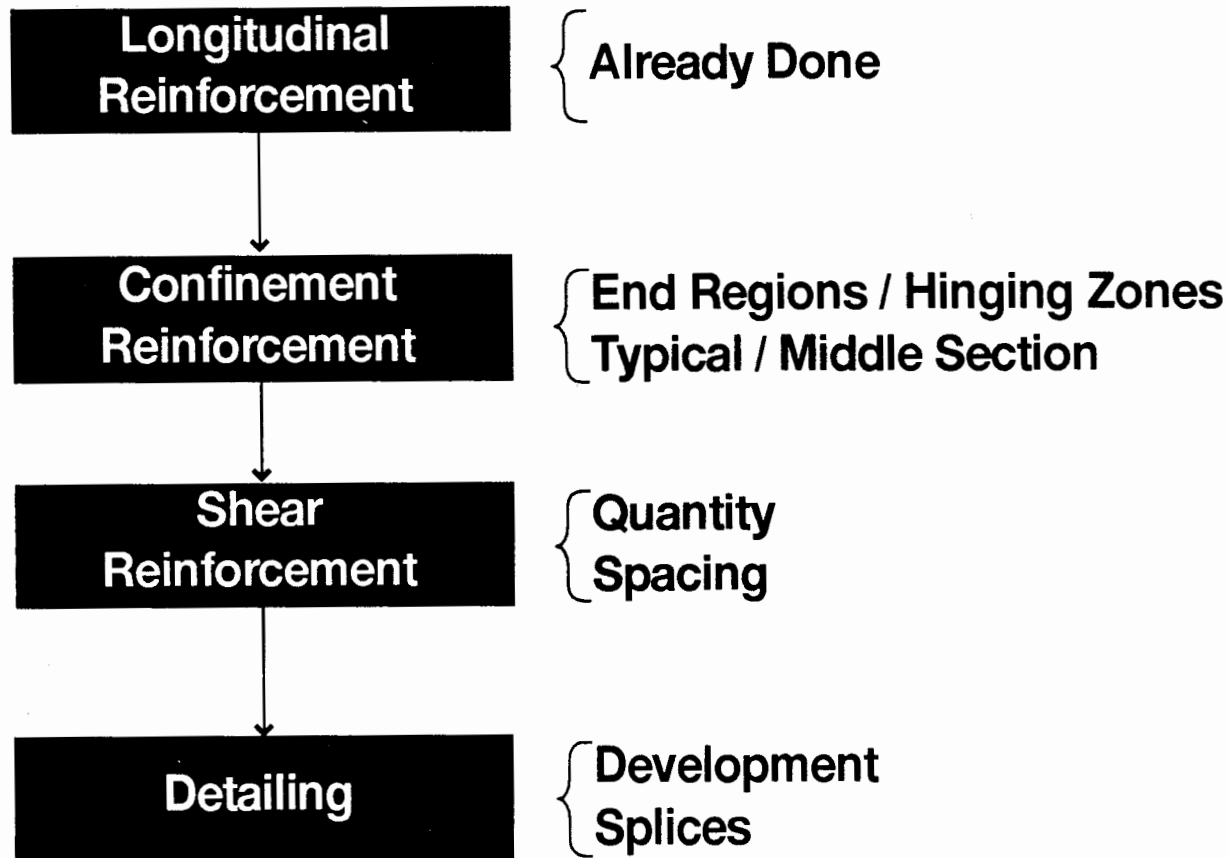
## Session 4

---

- Elastic Forces —→ Design Forces  
(Including Column Flexural Design)
- **Complete Column Design**
- Design Column Footings
- Abutment Issues

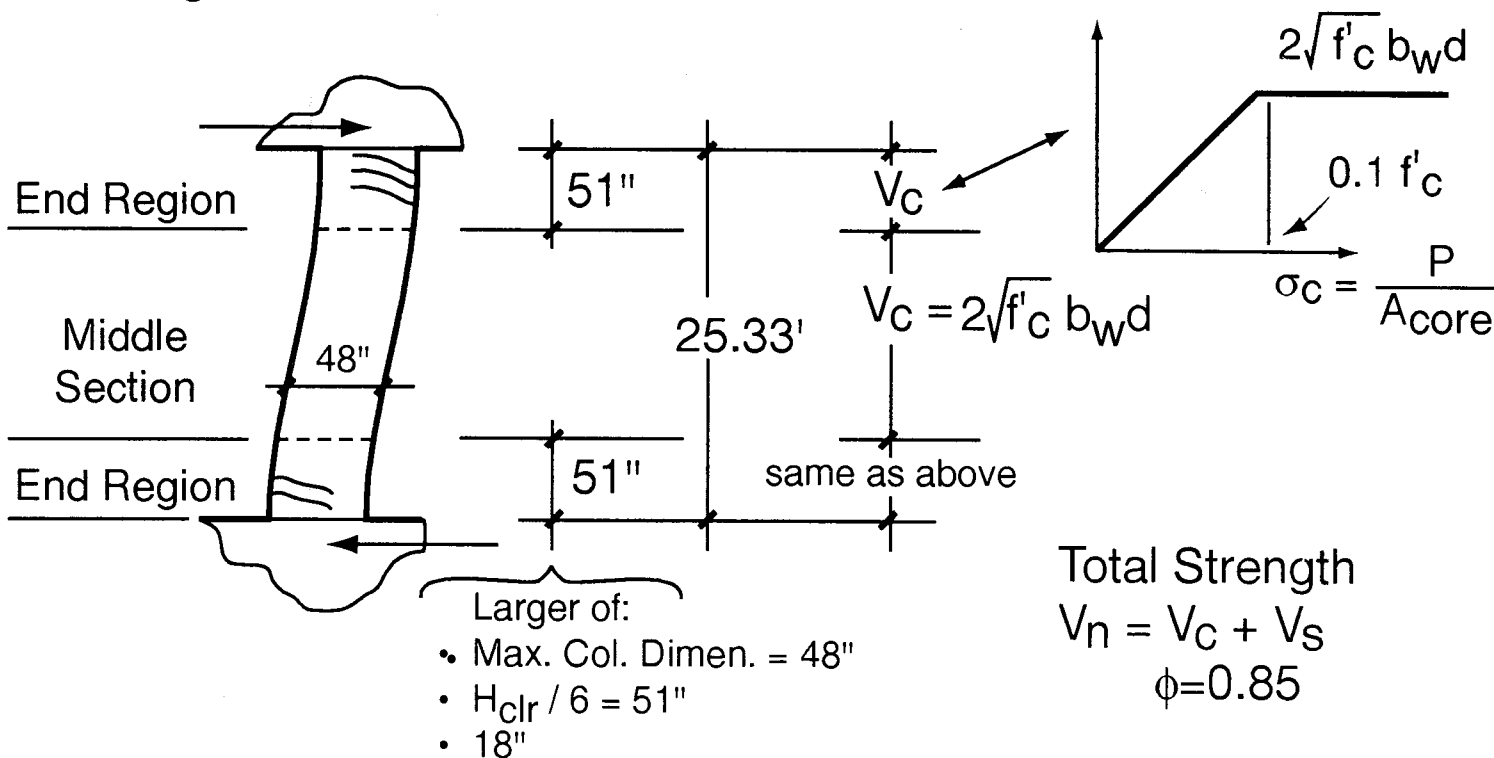
# Column Design (SPC C and D)

---



# Shear Strength

## • Strength – Two Zones:





# Confined Plastic Hinge Zone

## • Spirals

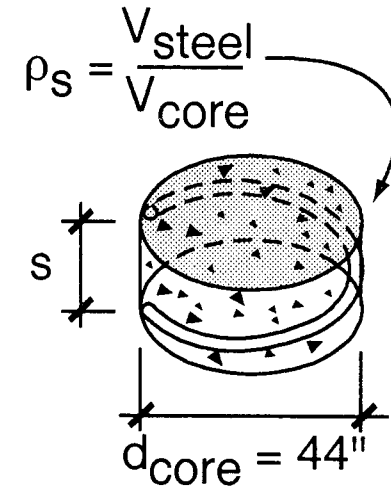
$$\rho_s = 0.45 \left( \frac{A_{\text{gross}}}{A_{\text{core}}} - 1 \right) \frac{f'_c}{f_{yh}} = 0.0057$$

$$\text{Minimum } \rho_s \geq 0.12 \frac{f'_c}{f_{yh}} = 0.008 \text{ Controls}$$

$$\text{Try } s = 3.5'' \text{ (} s \leq d_c/4 = 12'' \text{ and } s \leq 4'')$$

$$A_{sp} = \frac{\rho_s s d_{\text{core}}}{4d_s} = \frac{0.008 (3.5) 44^2}{4(44-0.625)} = 0.31 \text{ in}^2 \text{ Single Leg}$$

**Use #5 Spiral at 3.5'' Pitch for End Region**



## Shear Strength / End Region

---

- End Region

$$\frac{V_u}{\phi} = \begin{cases} \frac{426}{0.85} = 501 \text{ kip} & \text{Maximum Axial } P = 1432 \text{ kip} \\ \frac{385}{0.85} = 453 \text{ kip} & \text{Minimum Axial } P = 764 \text{ kip} \end{cases}$$

$$\sigma_c = \frac{P}{A_{\text{core}}} = \frac{764 \text{ kip}}{1521 \text{ in}^2} = 0.50 \text{ ksi} > 0.40 \text{ ksi} = 0.1 f'_c$$
$$V_c = 2 \sqrt{f'_c} b_w d$$

$V_c$  - Same for Either Axial Load

Use 501 kip as Required Shear Strength,  $\frac{V_u}{\phi}$

## Shear Strength / End Region (continued)

---

$$V_c = \frac{2\sqrt{4000}}{1000} (48)(37.2) = 226 \text{ kip}$$

$$V_s = \frac{V_u}{\phi} - V_c = 501 - 226 = 275 \text{ kip}$$

Use of Plastic Hinging Shear  
Prevents Brittle Shear Failure



For #5 Confinement Spiral at 3.5" Pitch  $V_s = 395 \text{ kip OK}$

## Shear Strength / Middle Section

---

$V_C = 226$  kip (Same as End Region)

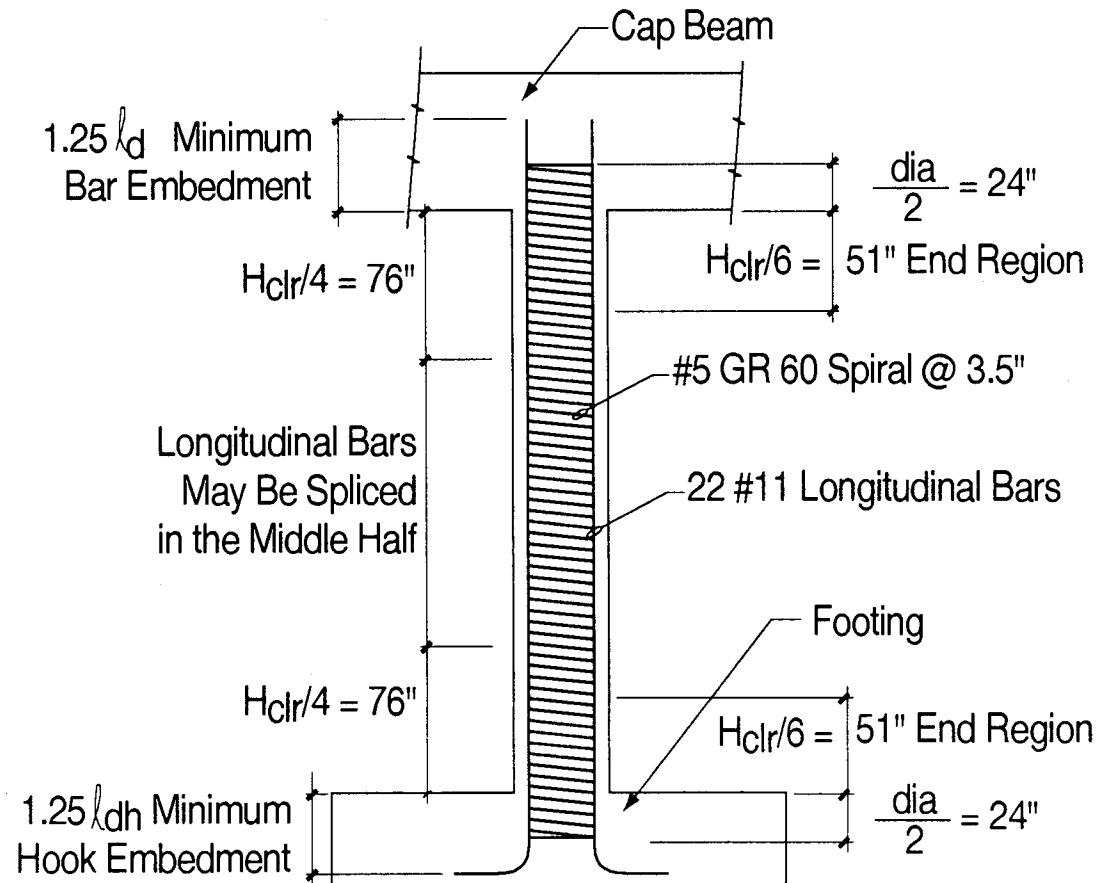
Required  $V_S = 275$  kip (Same as End Region)

#5 Spiral @ 3.5"       $V_S = 395$  kip  $> 275$  kip

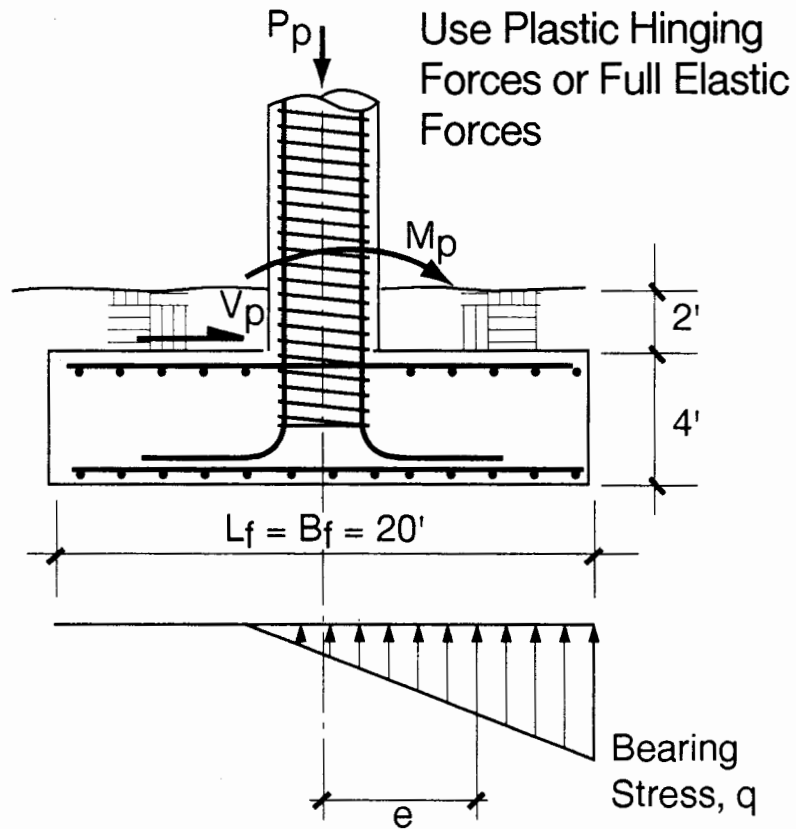
Could Open Up Spiral to 5" Pitch,  
but Keep at 3.5" to Avoid Construction Errors

Use #5 @ 3.5" Pitch throughout Column

# Column Reinforcement Details



# Spread Footings



## Issues

- **Soil Bearing Capacity**  
( $q_{ult} = 24$  ksf)
- **Overturning**  
(Uplift Over 1/2 Footing Dimension Is Permitted)
- **Sliding**  
( $\mu = 0.5$ , Neglect Passive)
- **Flexure in Footing**  
(Bottom and Top Steel)
- **Shear in Footing**  
(Stirrups?)

## Spread Footings (continued)

- **Use Plastic Hinging Forces**  
(Transverse Maximum and Minimum)

	Limit	Maximum P	Minimum P	OK?
Bearing, q Stress	24 ksf	9.8 ksf	8.8 ksf	✓
Overturning, e	6.7 ft	4.0 ft	5.9 ft	✓
Sliding, $\mu$ required	0.5	0.24	0.35	✓

Controls

**Could Reduce Footing Size Until  $e = b/3$**

# Session 4

---

- Elastic Forces —————> Design Forces  
(Including Column Flexural Design)
- Design Columns
- Design Column Footings
- Abutment Issues



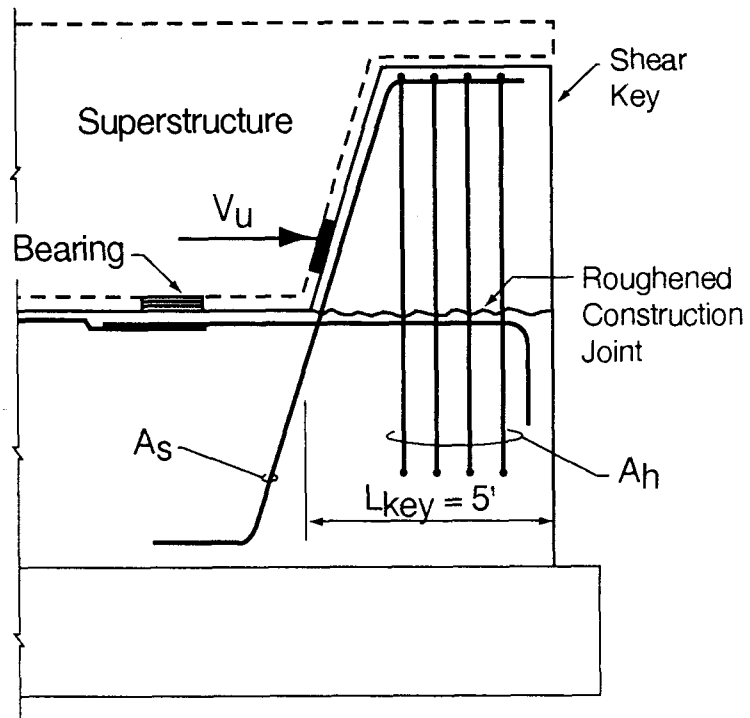
# Abutment Shear Key

- Use Shear Friction Design

$$V_u = 1596 \text{ kip (based on } R = 0.8)$$

$$L_{\text{key}} = 5 \text{ ft (based on } V_n < \begin{cases} 0.2 f'_c \\ 800 \text{ psi} \end{cases})$$

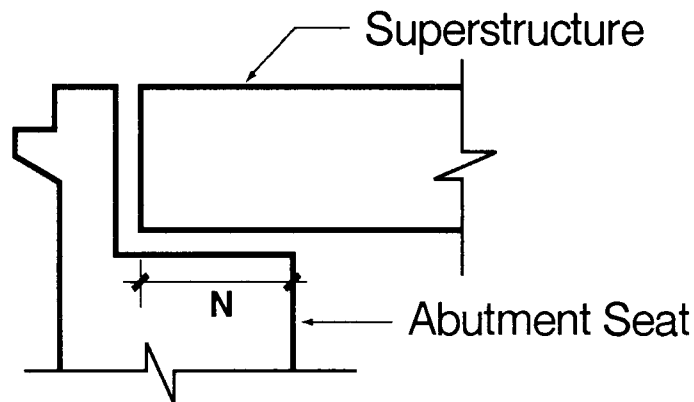
$$A_{vf} = 6.3 \text{ in}^2/\text{ft of Length}$$



# Displacements and Seat Widths

**For this Bridge Longitudinal Displacements Are Most Important**

- Analysis —  $\Delta_{\text{elastic}} = 0.24' (3'')$
- AASHTO — Seat Width, N, Prescriptive



$$N > \Delta_{\text{elastic}}$$

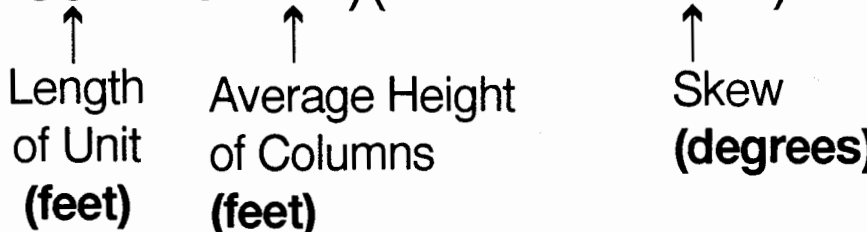
**Observed  $\Delta$ 's Larger than Simple Analysis Indicates**

## Seat Width

---

- **For SPC C**

$$N = (12'' + 0.03L + 0.12H)(1 + 0.000125S^2)$$

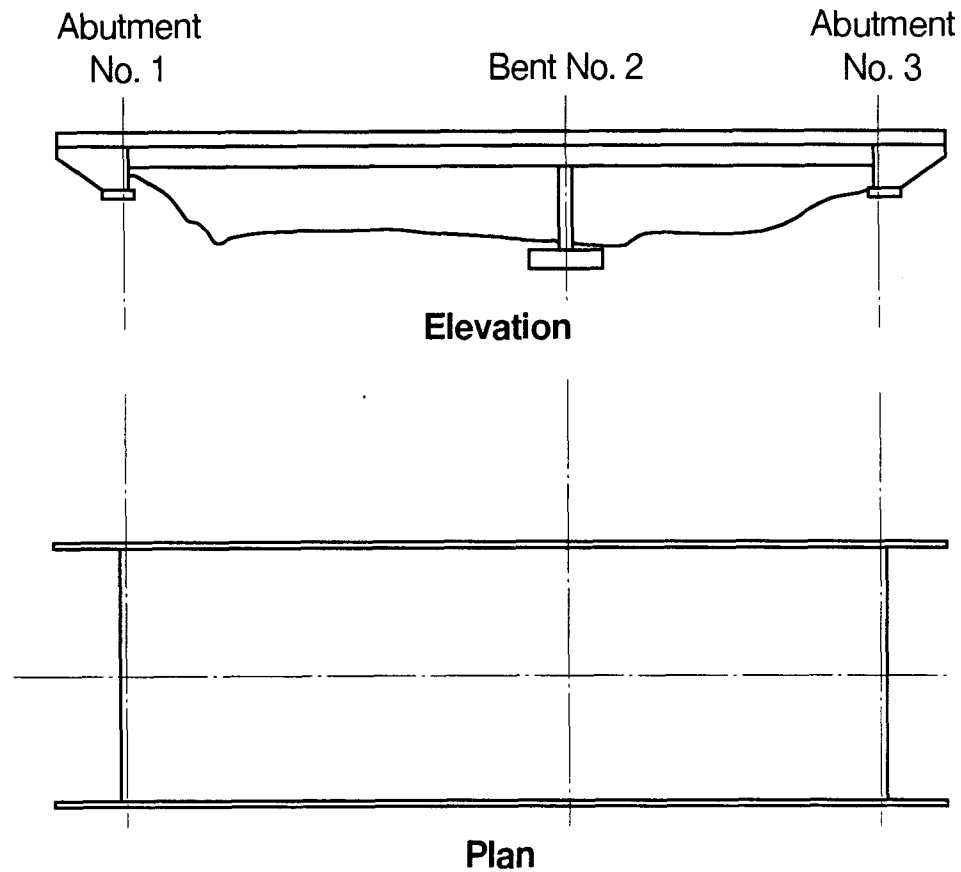
  
Length of Unit (feet)      Average Height of Columns (feet)      Skew (degrees)

$$N = (12'' + 0.03(242') + 0.12(27.34'))(1 + 0.000125(0^\circ)^2)$$

$$N = 22.5'' (1.88')$$

# Summary of Example Design

---



# **Session 5**

## **Modeling Guidelines**

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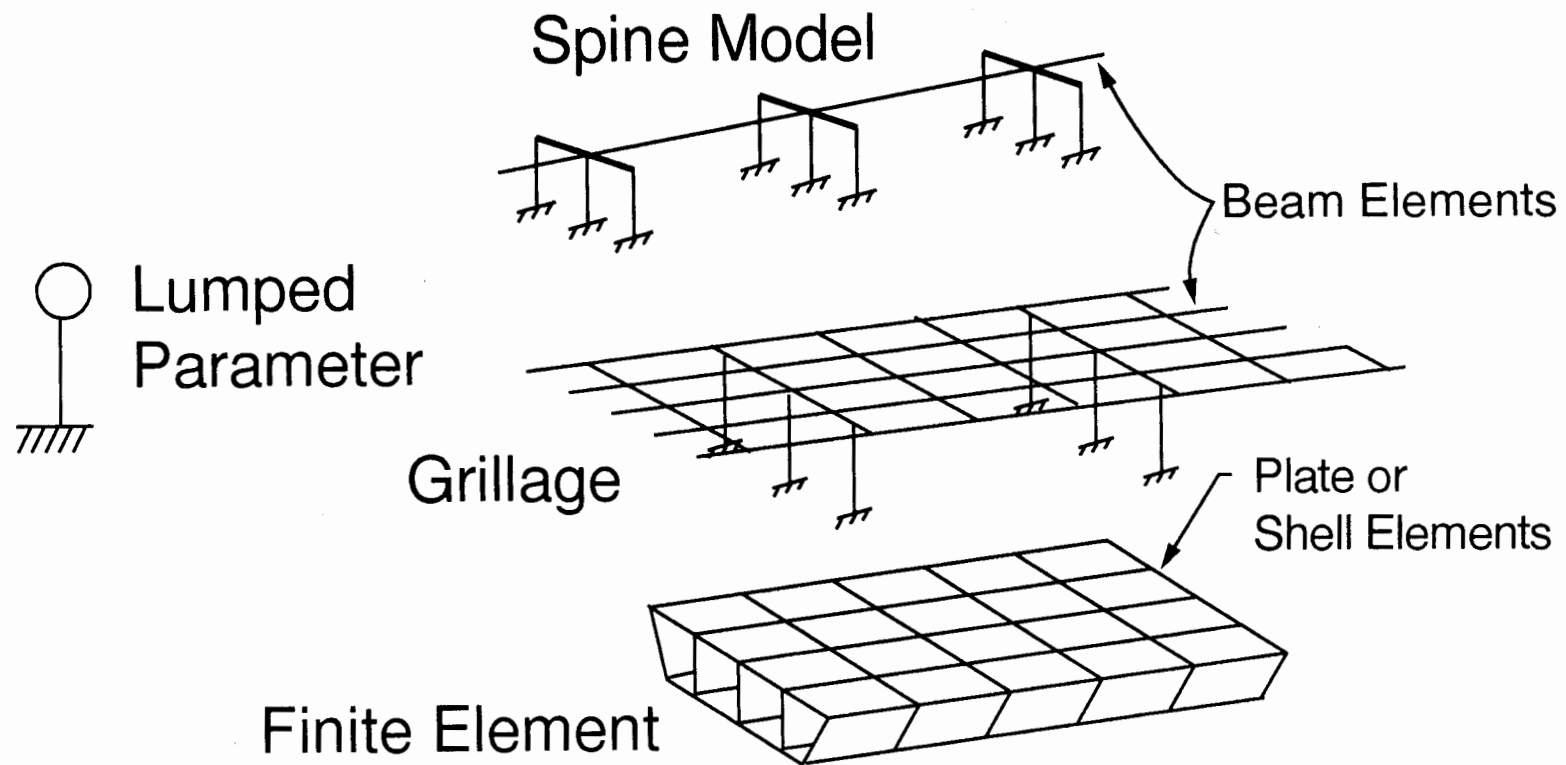
- **Types of Models**
- **Spine Model Considerations**
- **Properties**
- **Checking Modes**

# Modeling – General

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- **Analytical Models Should Include:**  
Stiffness Distribution of Bridge  
Mass Distribution of Bridge (for Multimode Analysis)
- **Commonly 3D Models Are Used**
- **Standard Computer Programs Are Used for Analysis**

# Types of Analysis Models



The diagram illustrates a mechanical system with multiple degrees of freedom. A horizontal beam is supported by a fixed support on the left and several vertical springs. The beam is divided into segments by points where vertical springs are attached. The right end of the beam is connected to a vertical spring. The diagram also shows a detail of a rotational spring, which is a spring that resists rotation, and a translational spring, which resists linear displacement. The labels 'Translational Spring' and 'Rotational Spring' are used to identify these components. The word 'Or' is placed between the two spring types, indicating that they are alternative representations of the same physical concept.

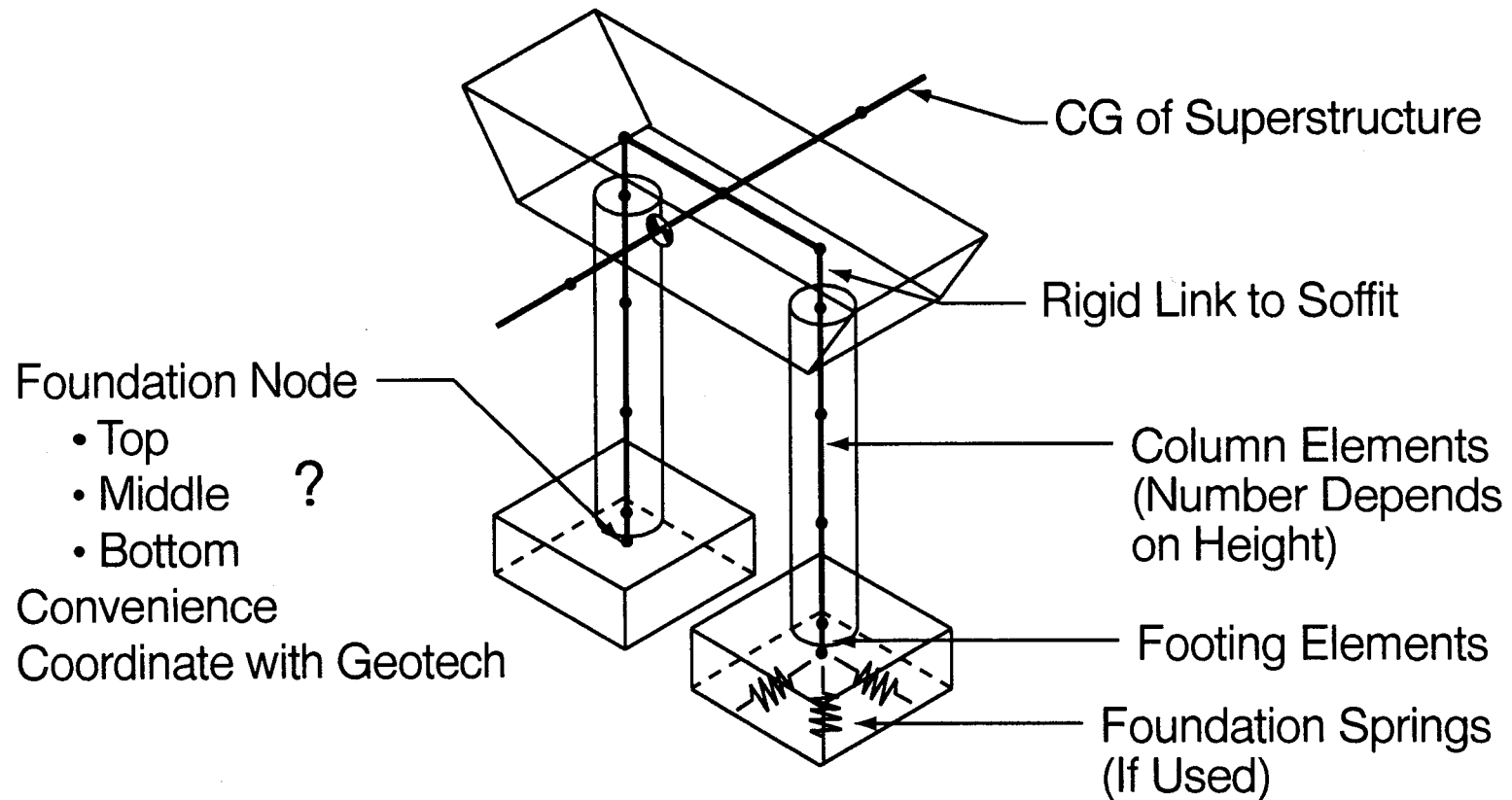
- Session 5 Page 4 of 39

## Seismic Bridge Design Applications

25 April 1996, NHI Course Code No. 13063



# Spine Model – Geometry Issues



# Properties

---

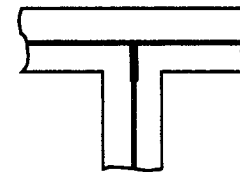
**Stiffness**  
(e.g. Concrete)

$E \sim 57\sqrt{f'_c}$  Concrete

$I \sim \begin{cases} I_{\text{gross}} & \dots \text{Maximum Forces} \\ I_{\text{effective}} & \dots \text{Maximum Displacement} \\ I_{\text{transformed}} & \dots \text{Too High} \end{cases}$

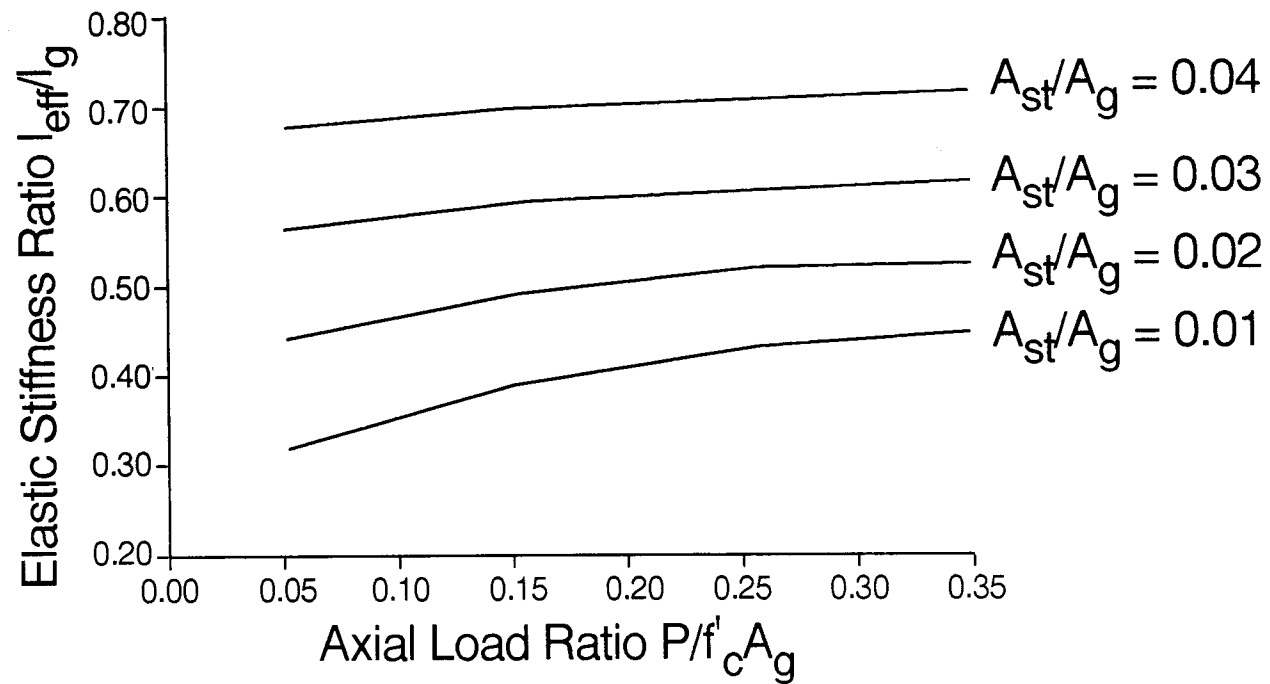
$A \sim$  Area

$L \sim$  Adjust for Joint Stiffness



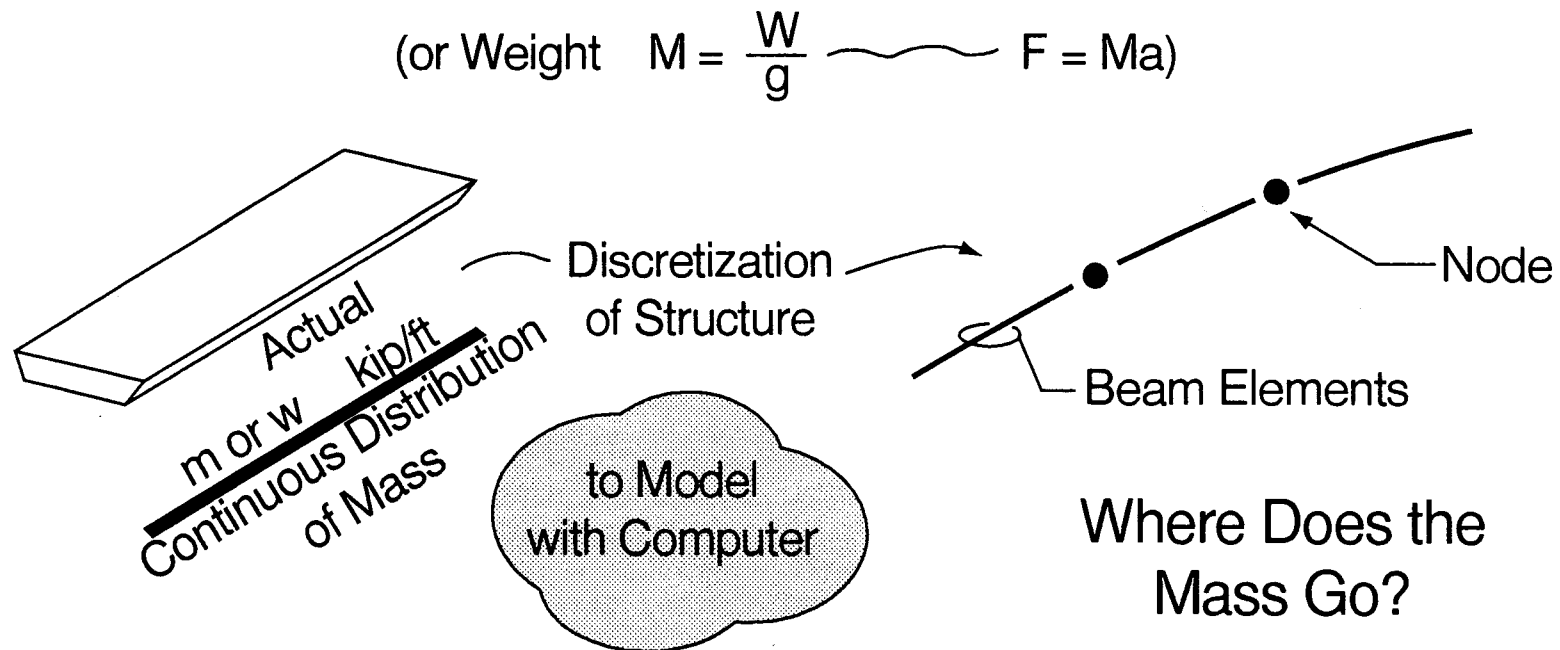
# Effective Moment of Inertia – RC Columns

## Circular Sections



Priestley, Seible, and Calvi

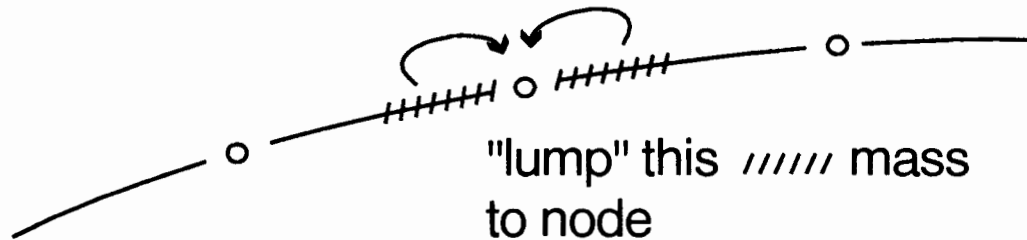
# Mass Distribution



**Superstructure Mass Usually Most Significant**

## Mass Distribution (continued)

---

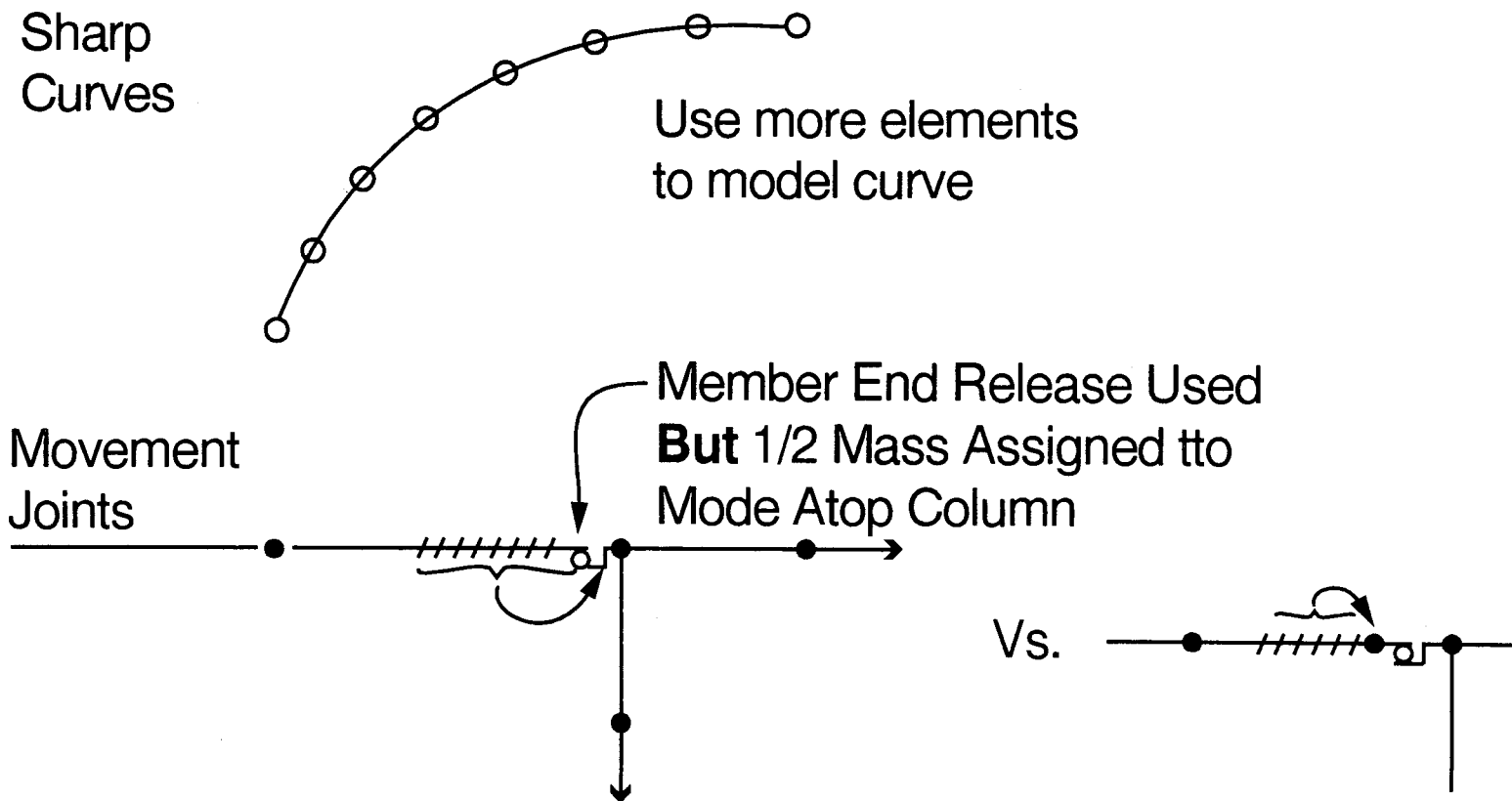


(example)

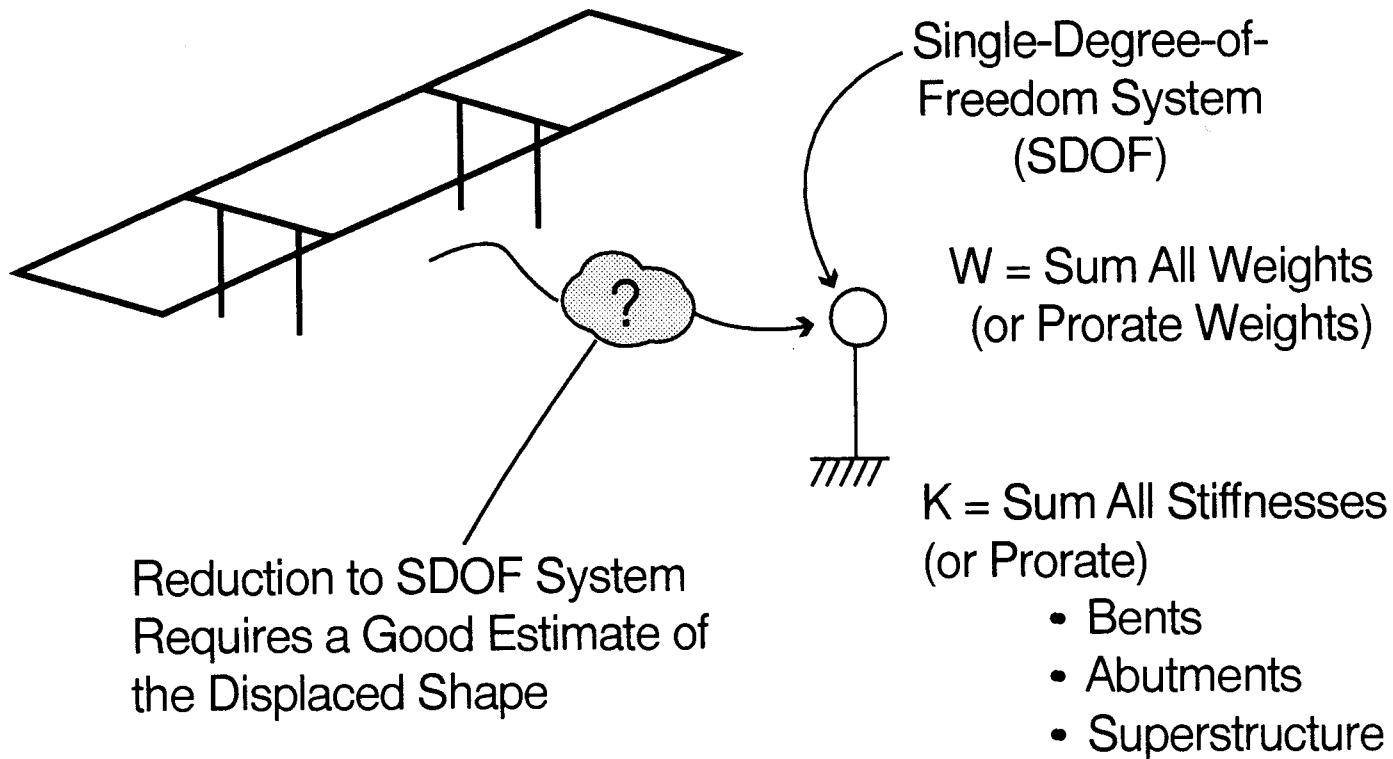
Options:

- |                  |   |  |
|------------------|---|--|
| members          | → | • Specify density, $\sigma$ , and area, $A$ , of element |
| traffic barriers | → | • Specify weight per length, $w$                         |
|                  |   | • Specify weight for node, $W$ , directly                |
| diaphragms       | → |  |

# Special Considerations



# Lumped Parameter – Checking and Simple Cases



# **Session 5**

## **Foundation Modeling**

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- **Structural / Geotech Relationship**
- **Soil Behavior**
- **Perspective — Using Soil Springs**
- **Modeling the Soil**



# Foundation Analysis and Design Issues

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- **This Session**

General Behavior Concepts

Simple Concepts for including Flexibility  
of Foundations

- **Next Seminar**

More Detailed Analysis Techniques

Discussion of Design Issues

## Structural Engineer

## Geotechnical Engineer

---

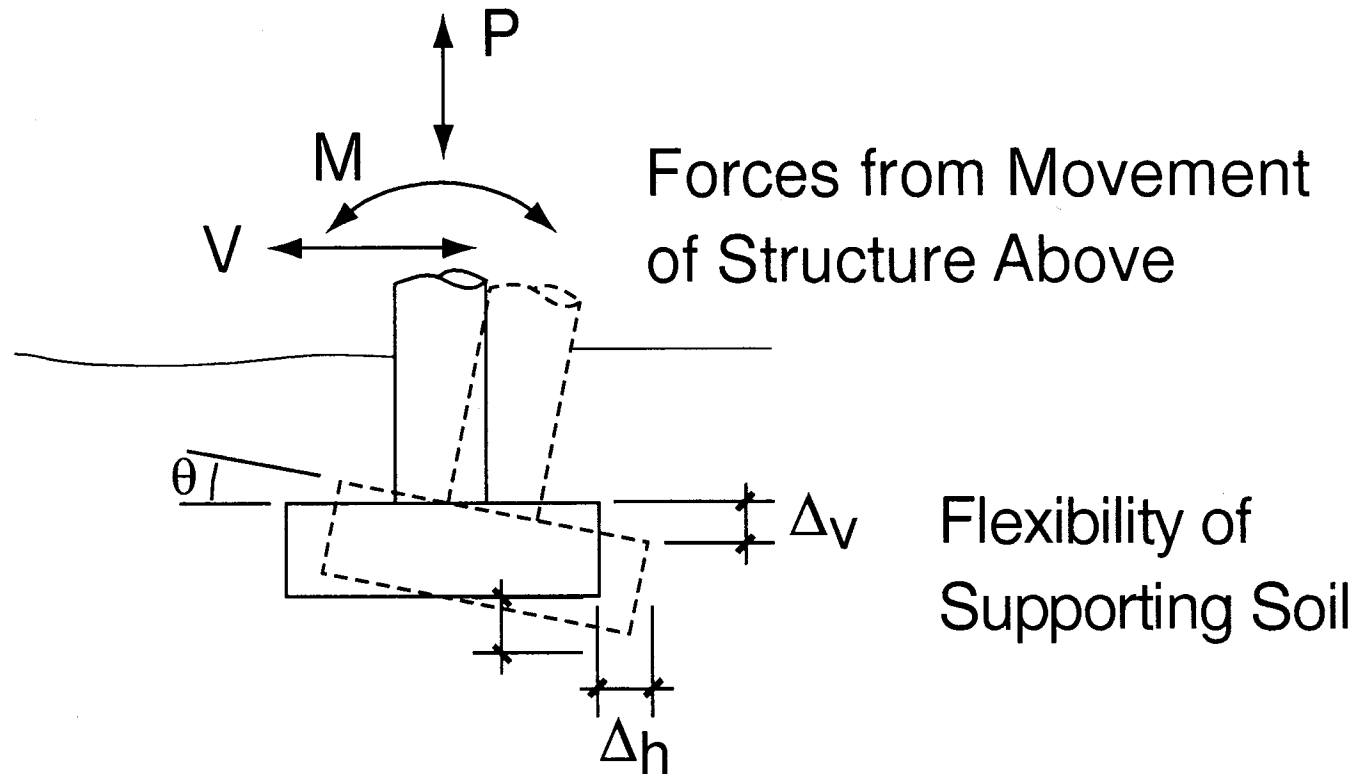
### Needs

- Hazard / Spectra
- Foundation Concepts
- Soil Properties
- Soil Capacities
- Modeling Assistance
- Liquefaction Assessment

### Needs

- Substructure Types
- Soil Load Magnitudes
- Displacements
- Comparison Types
  - Service
  - Ultimate

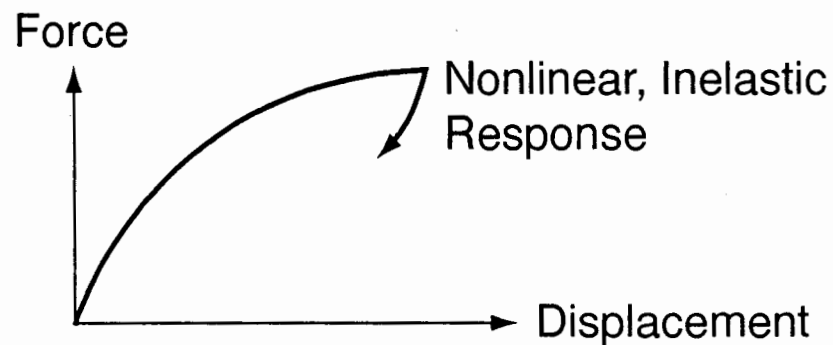
# Foundation Behavior – Spread Footing



# Foundation Behavior (continued)

---

## Stiffness Effects



## Damping Effects

Damping in Soil  $\rightarrow$  Energy Dissipation  
(e.g. 'Radiation Damping')

## Mass Effects

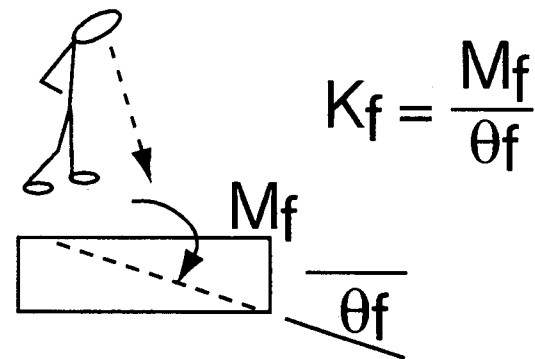
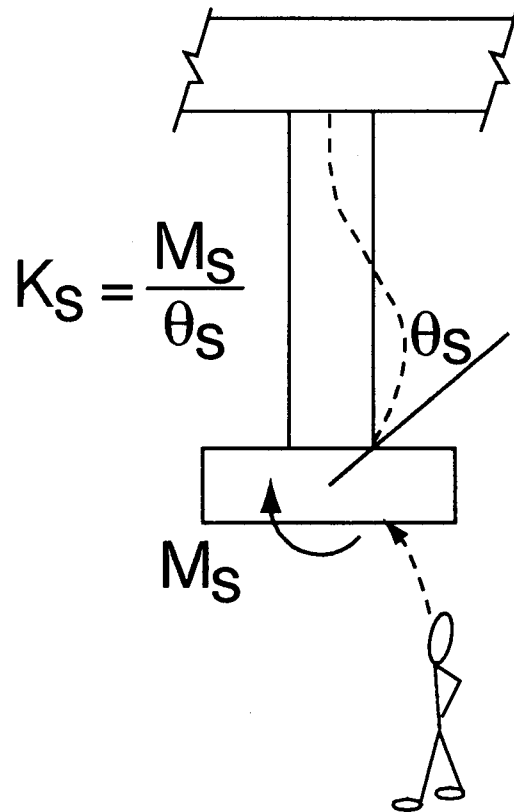
Soil Mass Affects Response

# Using Spring Supports

---

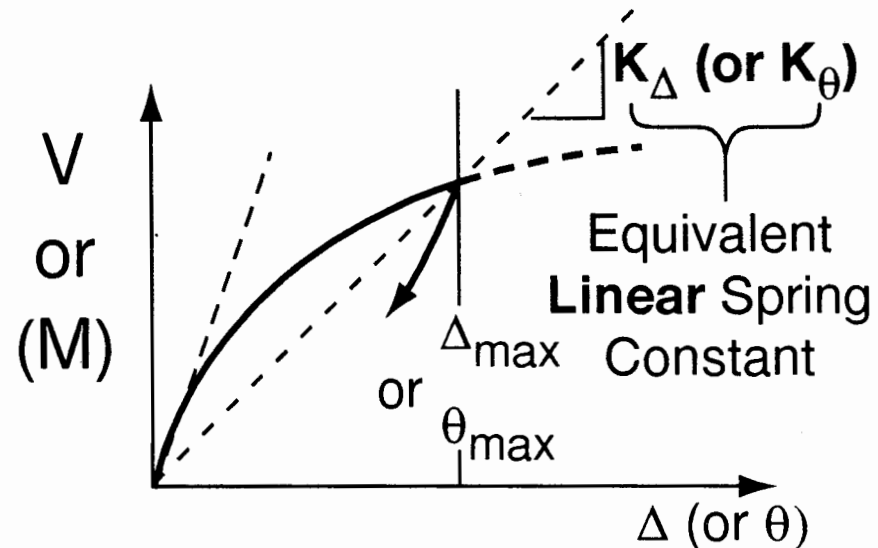
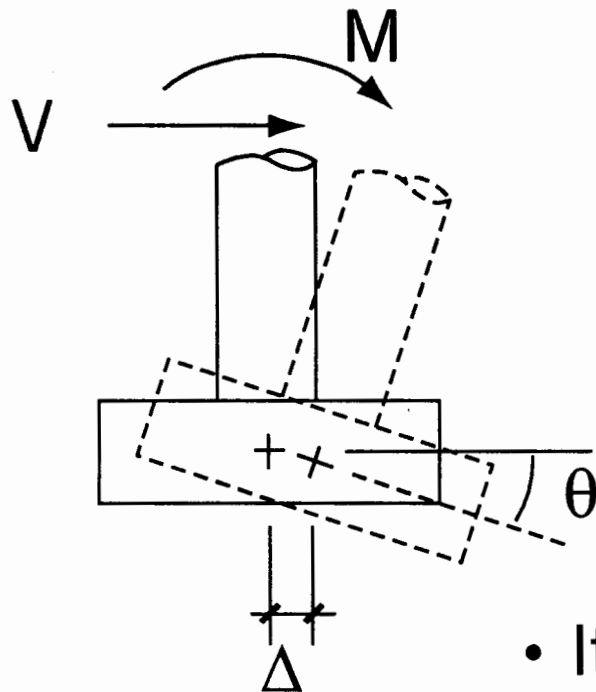
- When      **Refinement** of Seismic Analysis  
(**After** Bounding Analyses with Fixed  
or Free Supports)
- Why      Soil Flexibility Is Significant Relative to Structure
- How      Equivalent Linear Springs
- Accuracy      Actual Spring Constant Not as Important as  
Presence of Spring Itself

# Stiffness – Structure vs. Foundation



$K_f \gg K_S \rightarrow$  Fixed  
 $K_f \ll K_S \rightarrow$  Pinned  
 $K_f \approx K_S \rightarrow$  Springs (or Bound)

# Soil Response May Be Nonlinear

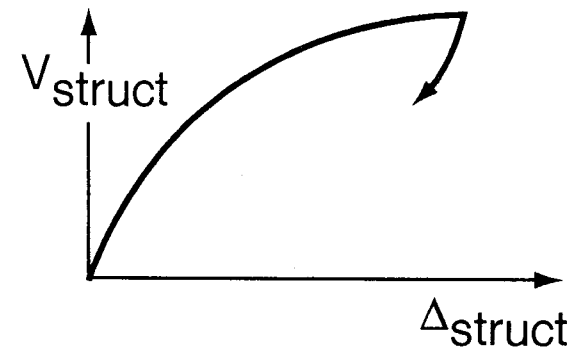


- Iteratively Determine  $K$ 's or 'Bound' Response

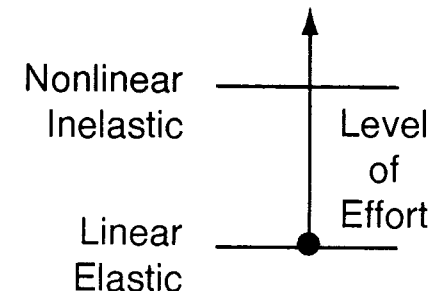
# Perspective on Nonlinear Behavior

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- Recall that Structure Nonlinear (Inelastic/Yielding) Behavior Is Expected

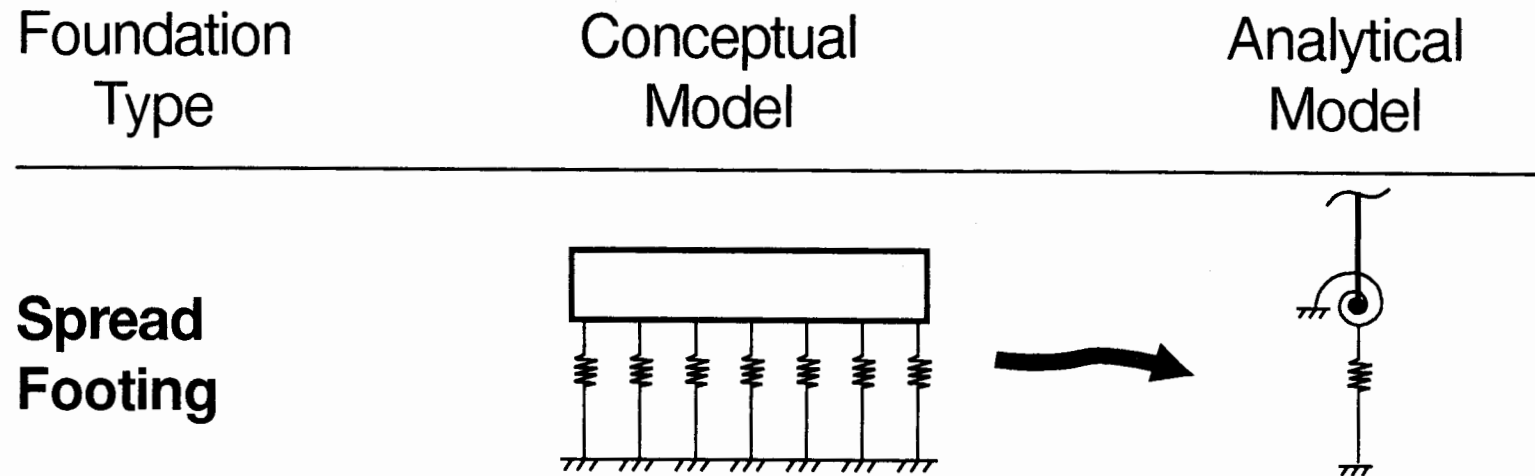


- Reasonable to Allow Some Nonlinear Soil Response
- Reasonable to Use **Elastic** Analysis





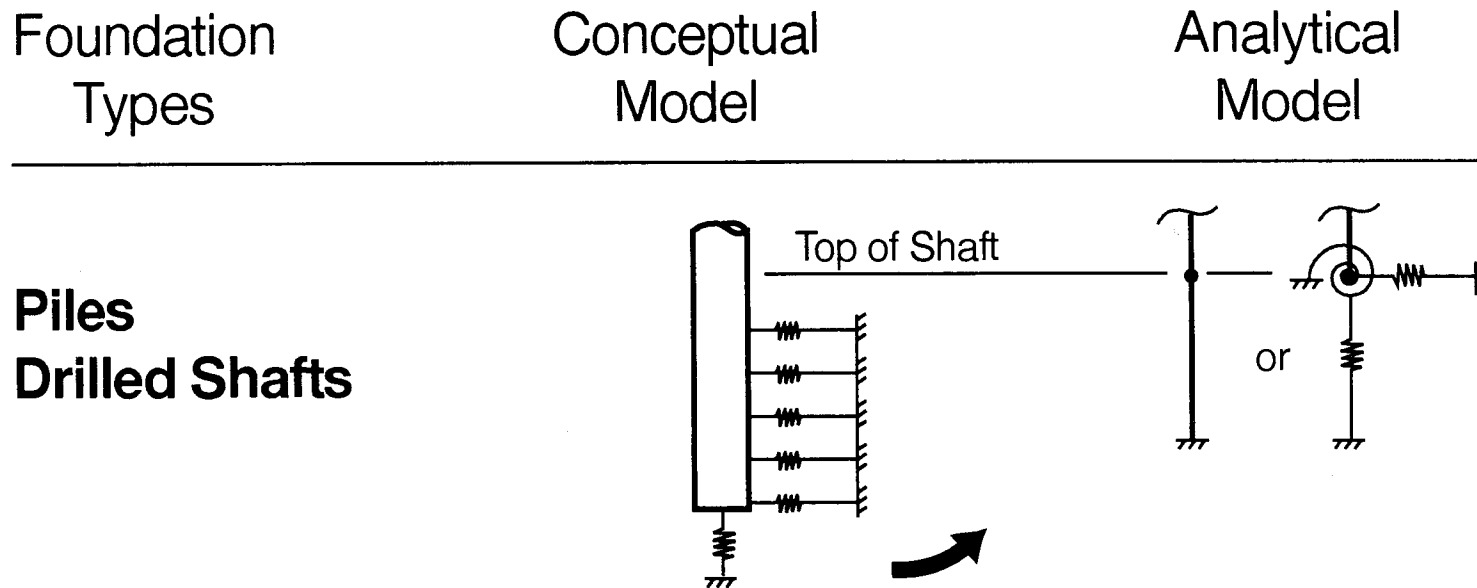
# Modeling Soil Flexibility



Reference: Bowles, 1988  
FHWA - IP-87-6

Design Examples: 1, 2, 4

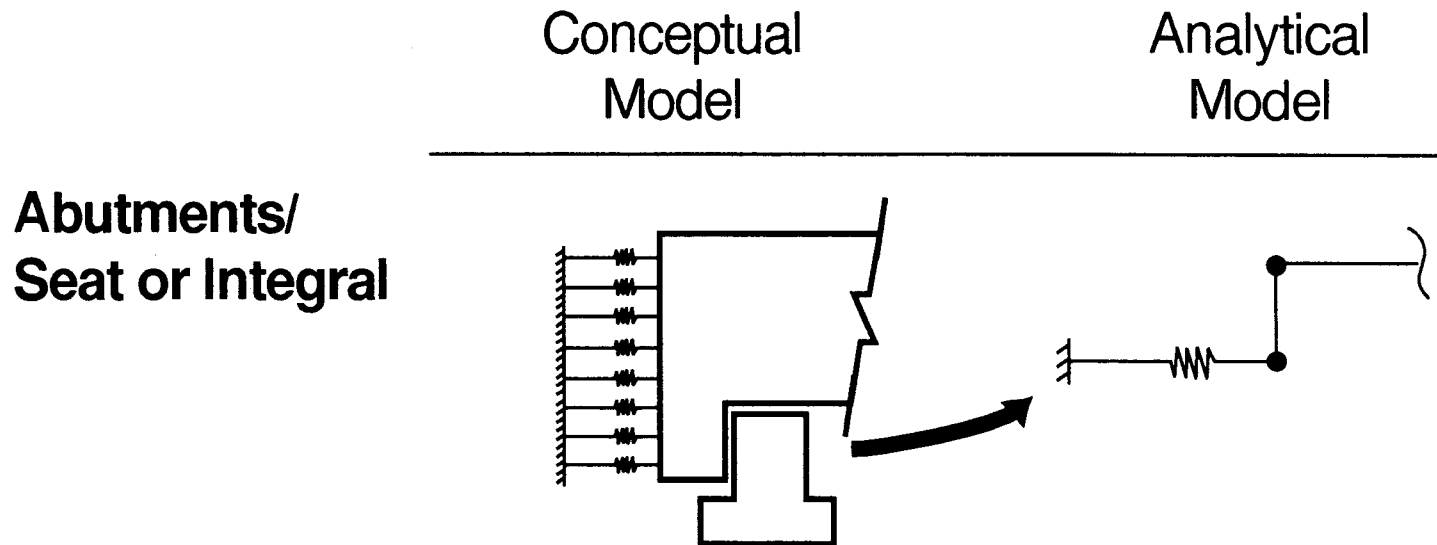
# Modeling Soil Flexibility (continued)



Reference: NAVFAC DM 7.02, 1986  
FHWA - IP-87-6

Design Examples: 5, 6

# Modeling Soil Flexibility (continued)



Reference: Caltrans 1995

Design Examples: 1, 3, 5, 6

# **Session 5**

## **Multimode Dynamic Analysis**

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- **Definition**
- **Using Computational Tools**

Input Data

Process Flow

Decisions

# Multimode Dynamic Analysis

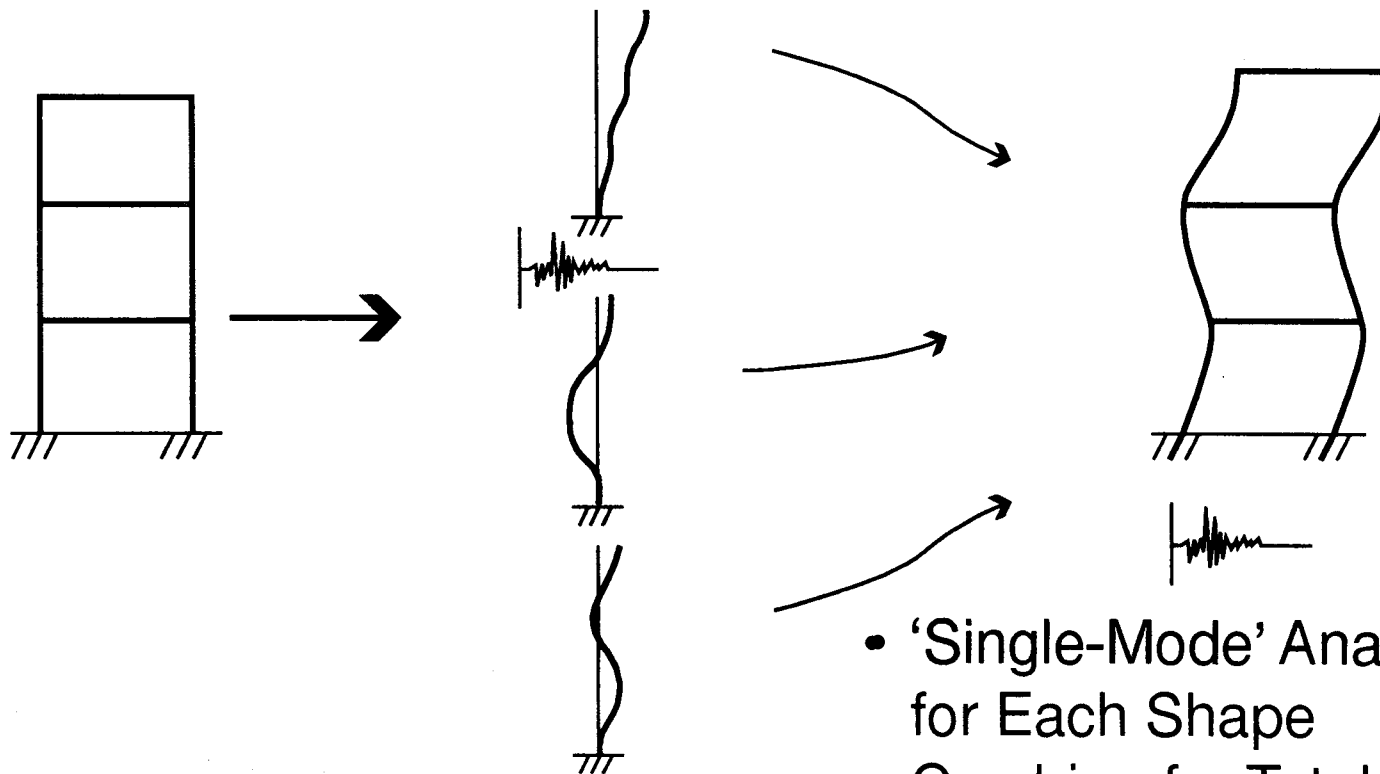
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- **What Is It?**

Superimpose Individual Mode Responses  
to Estimate Structural Response

(Similar to Using Base Colors to Make Paint)

# Multimode Concepts



- 'Single-Mode' Analysis for Each Shape
- Combine for Total

# Multimode Dynamic Analysis

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- **Why?**

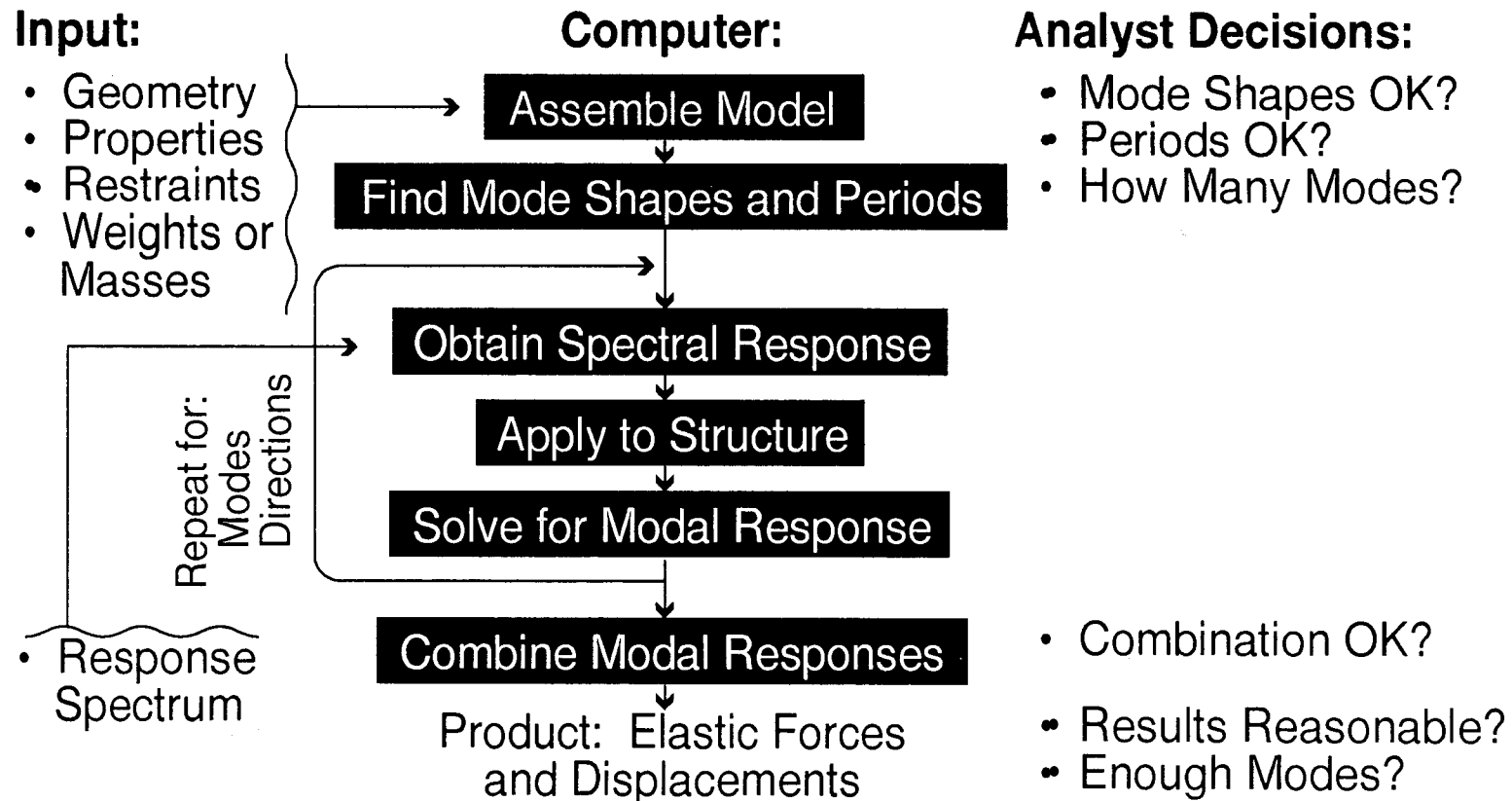
Gives Reasonable Estimate of Forces  
and Displacements

Especially Helpful for Complex and/or  
Irregular Structures

- **Limits?**

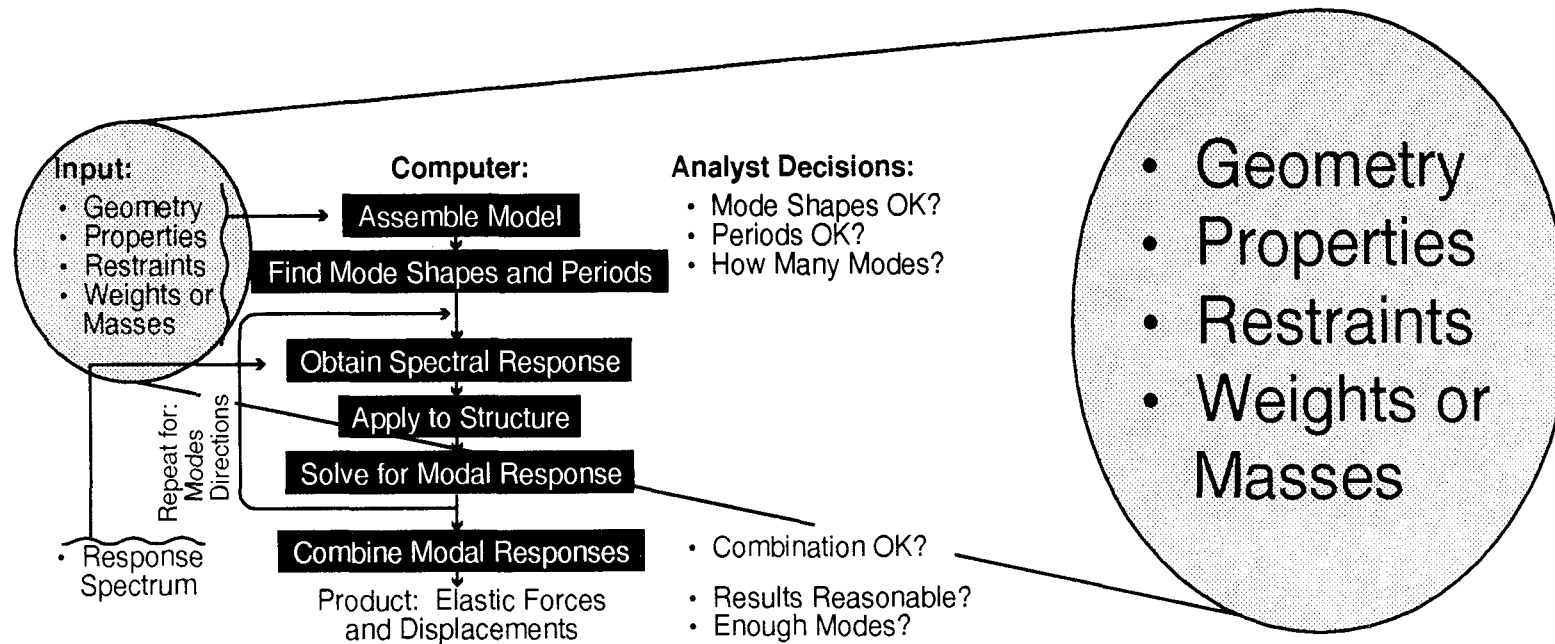
Applies Only to Linear-Elastic (Non-Yielding)  
Structures

# Multimode Dynamic Analysis

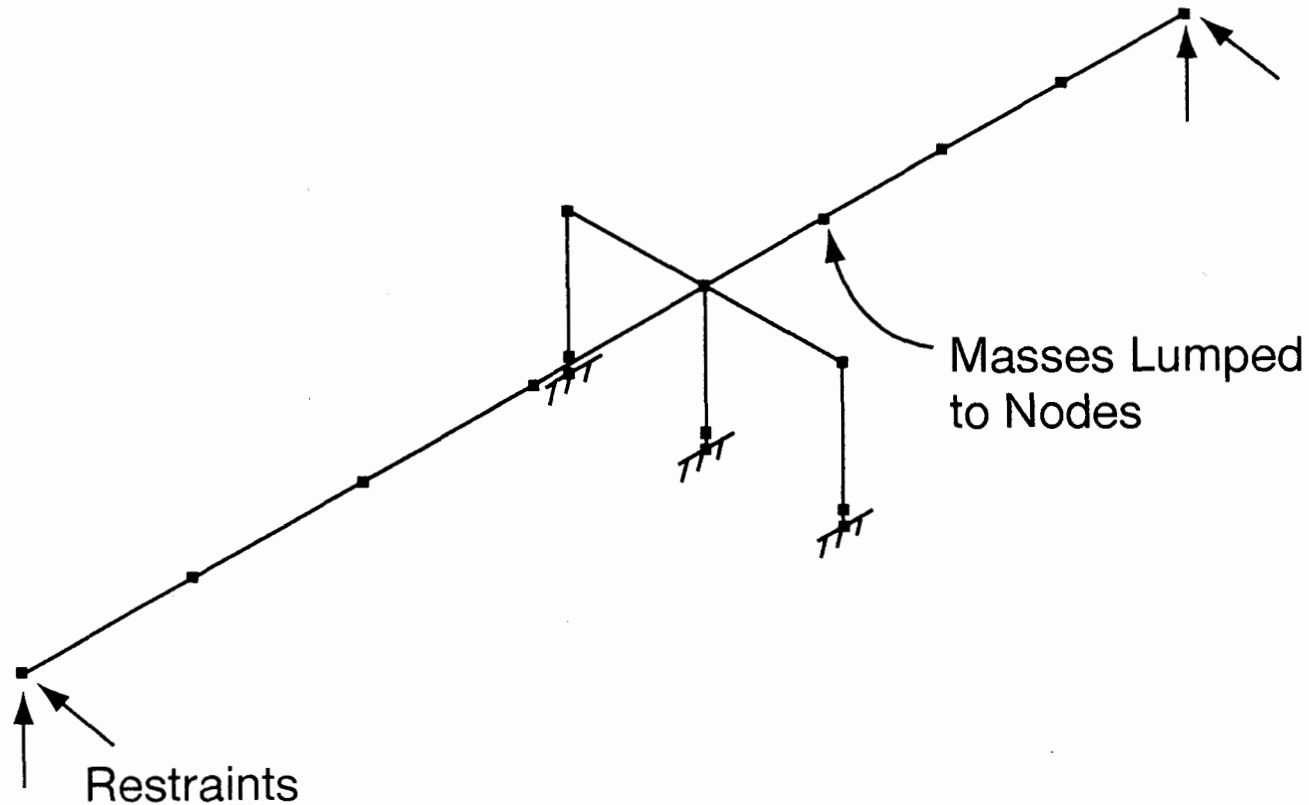




# Multimode Dynamic Analysis



# Example Bridge – Spine Model



# Mode Shape Terminology – 2D vs. 3D

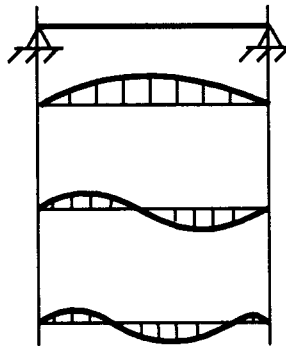
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2D

1st Mode

2nd Mode

3rd Mode



... Fundamental Mode

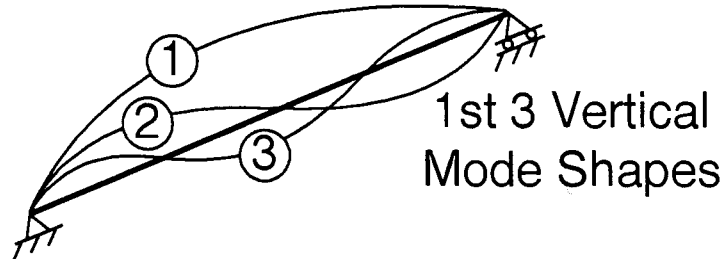
... Higher Modes

3D '1st, 2nd, 3rd' in Each Orthogonal Direction

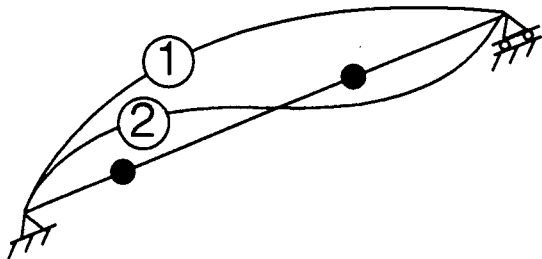
# Breaking Structure into 'Discrete' Elements

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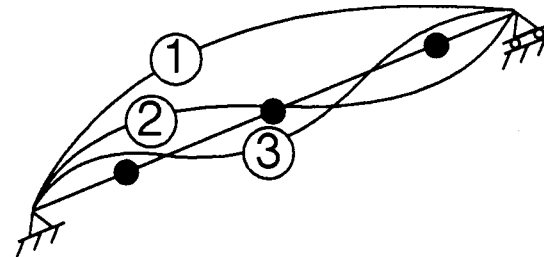
Distributed Mass  
Structure (Actual)



Two Nodes:  
(2 Modes / Direction)



Three Nodes:  
(3 Modes / Direction)

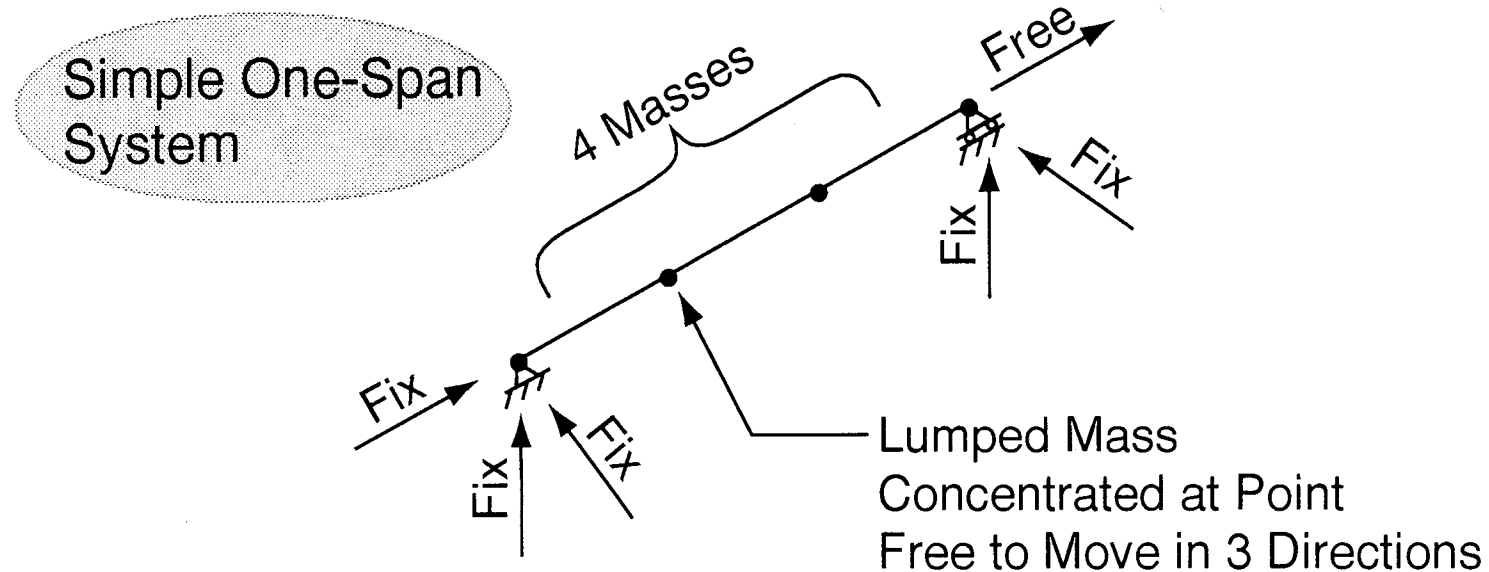


- More Nodes → Refinement of Forces
- AASHTO / Use 4 Elements (3 Nodes) per Span

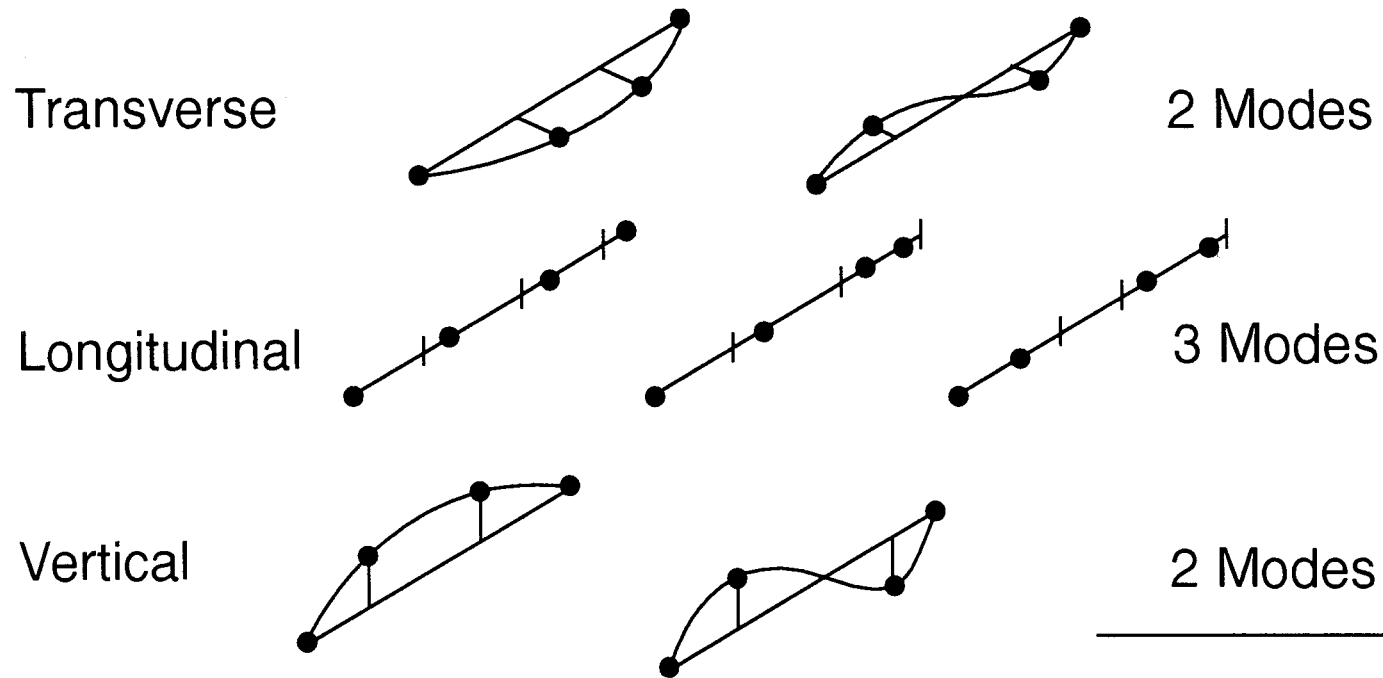
# Number of Modes Possible – 3D

## Number of Modes Depends on:

- Number of Masses
- Boundary Conditions / Restraints



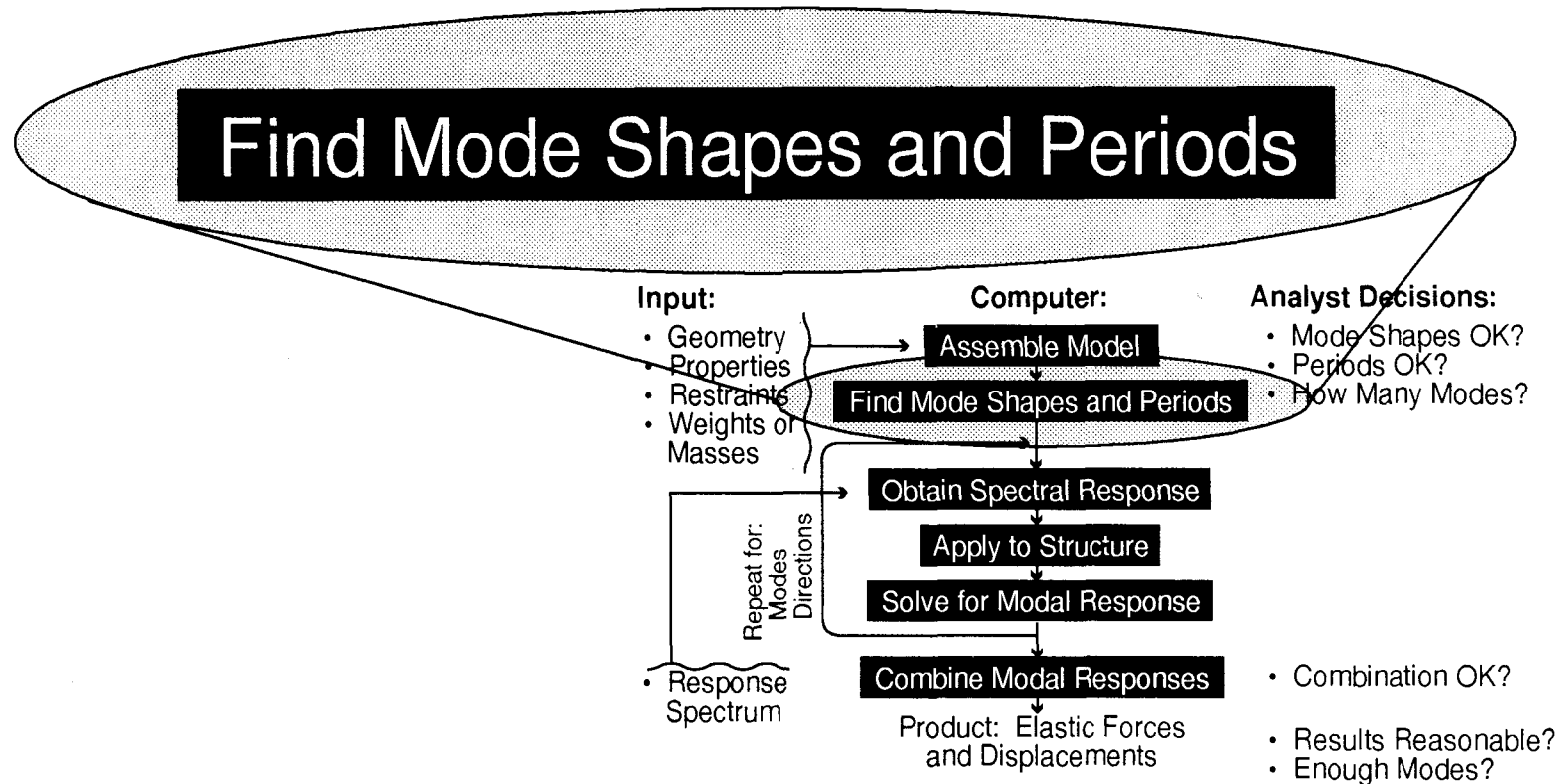
# Number of Modes Possible (continued)



$$(3n_{\text{masses}} - \text{Restrains}) = 7 \text{ Modes}$$

4 ——— 5

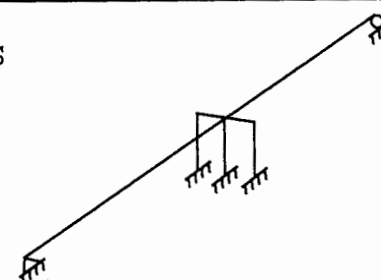
# Multimode Dynamic Analysis



# Example Results – Ordering of Modes

## E I G E N   S Y S T E M   P A R A M E T E R S

NUMBER OF EQUATIONS	=	78
NUMBER OF MASSES	=	38
NUMBER OF VALUES TO BE EVALUATED	=	15
SIZE OF SUBSPACE	=	19



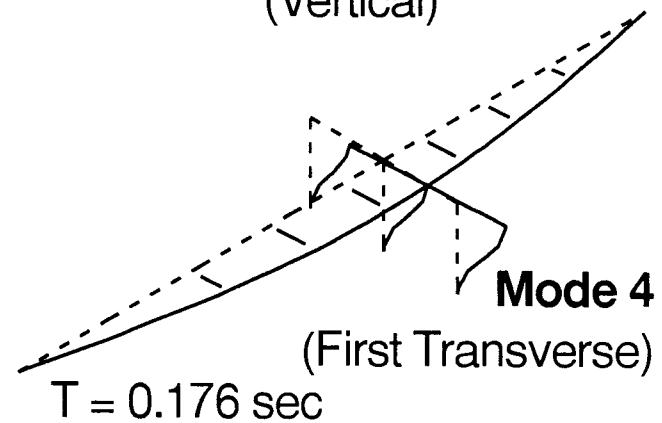
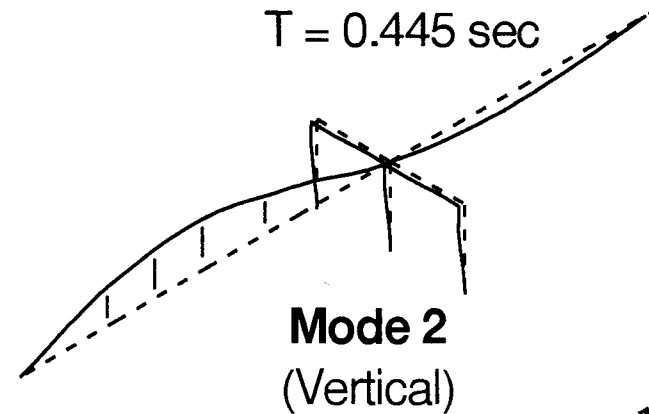
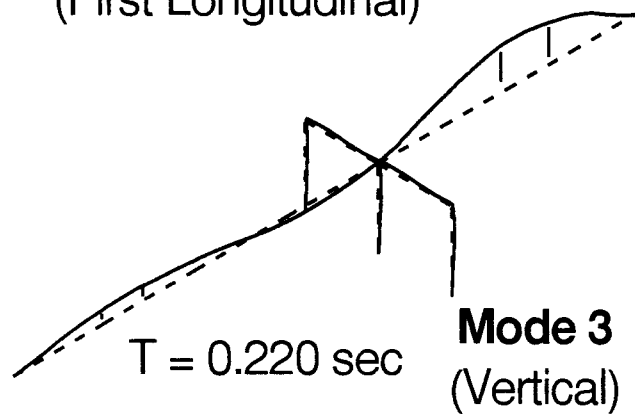
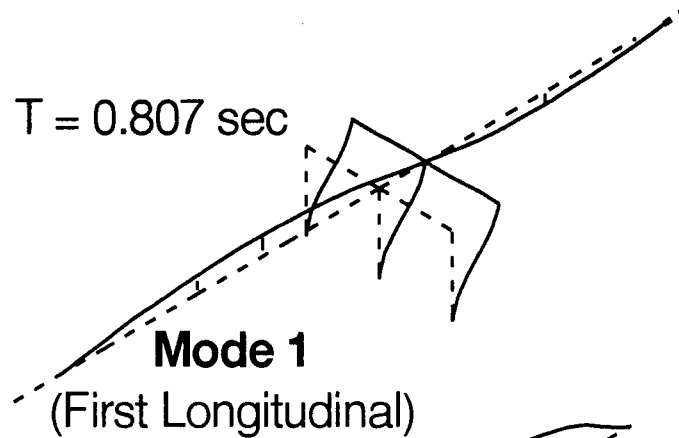
## E I G E N V A L U E S   A N D   F R E Q U E N C I E S

MODE NUMBER	EIGENVALUE (RAD/SEC)**2	CIRCULAR FREQ (RAD/SEC)	FREQUENCY (CYCLES/SEC)	PERIOD (SEC)
1	0.606219E+02	0.778601E+01	1.239182	0.806984
2	0.199010E+03	0.141071E+02	2.245216	0.445392
3	0.818561E+03	0.286105E+02	4.553502	0.219611
4	0.128067E+04	0.357865E+02	5.695596	0.175574
5	0.244260E+04	0.494226E+02	7.865859	0.127132
6	0.840641E+04	0.916865E+02	14.592353	0.068529
7	0.957479E+04	0.978508E+02	15.573445	0.064212
8	0.174203E+05	0.131986E+03	21.006200	0.047605
9	0.191682E+05	0.138449E+03	22.034859	0.045383
10	0.269107E+05	0.164045E+03	26.108526	0.038302
11	0.437236E+05	0.209102E+03	33.279597	0.030048
12	0.622932E+05	0.249586E+03	39.722864	0.025174
13	0.932711E+05	0.305403E+03	48.606414	0.020573
14	0.130158E+06	0.360774E+03	57.419016	0.017416
15	0.139134E+06	0.373006E+03	59.365809	0.016845

15 of 38  
Total

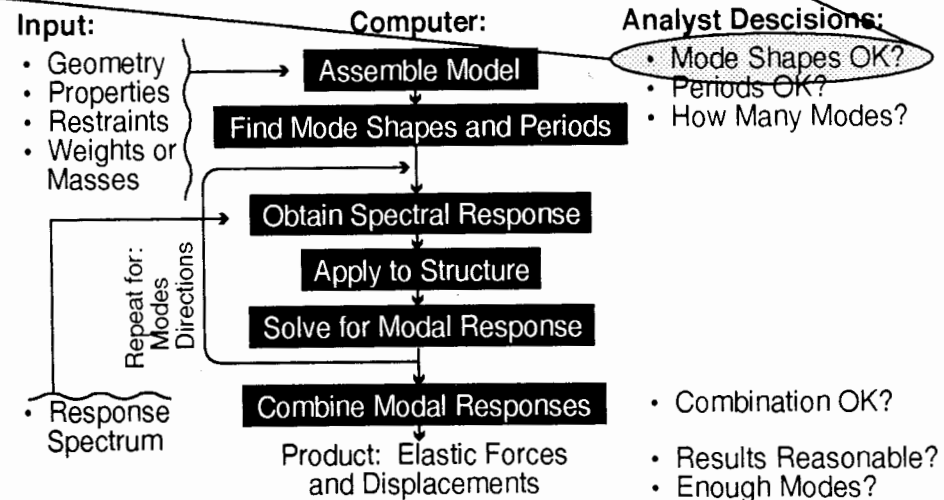


# Example Bridge – Mode Shapes



# Multimode Dynamic Analysis

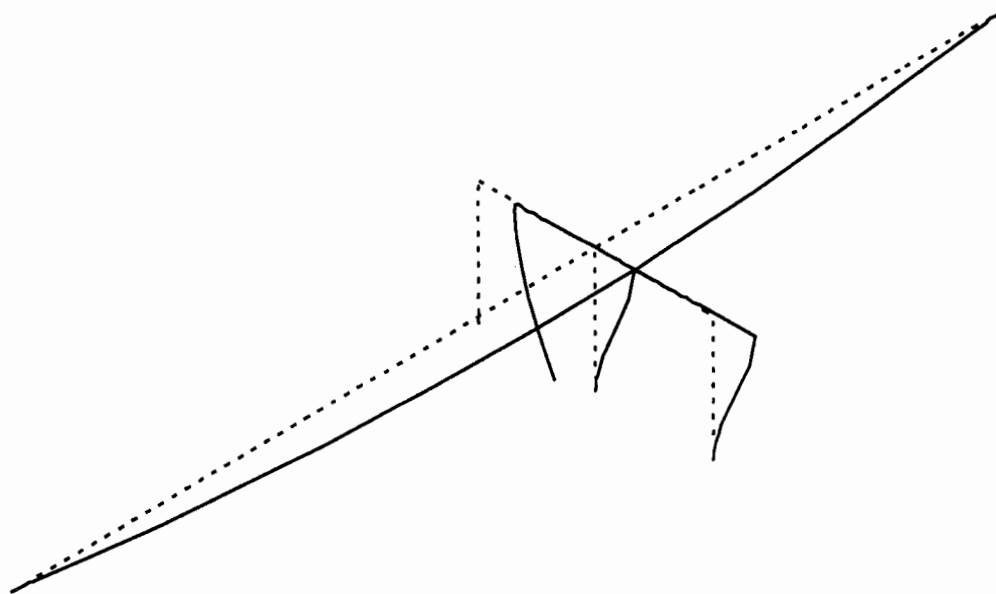
## • Mode Shapes OK?



# Checking Results with Mode Shapes

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- Inspect All Mode Shapes for Realism





# **Session 6**

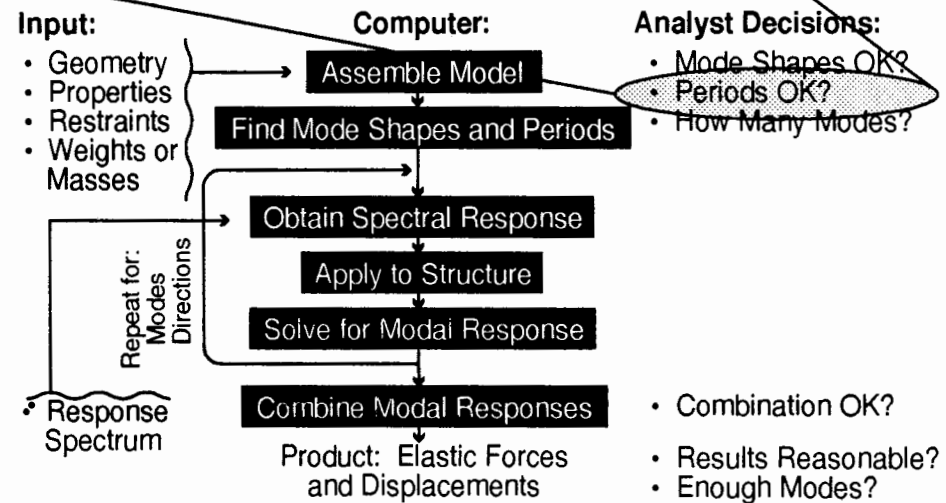
## **Multimode Dynamic Analysis**

---

- **Continuation of Session 5**

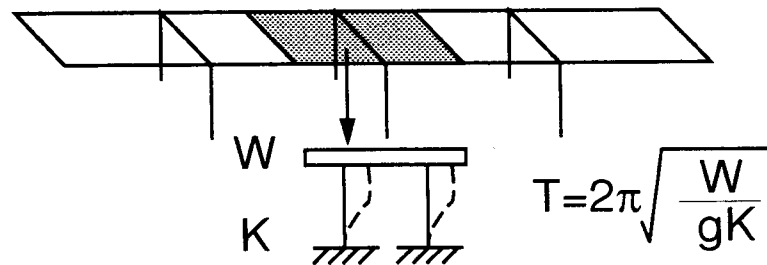
# Multimode Dynamic Analysis

## • Periods OK?

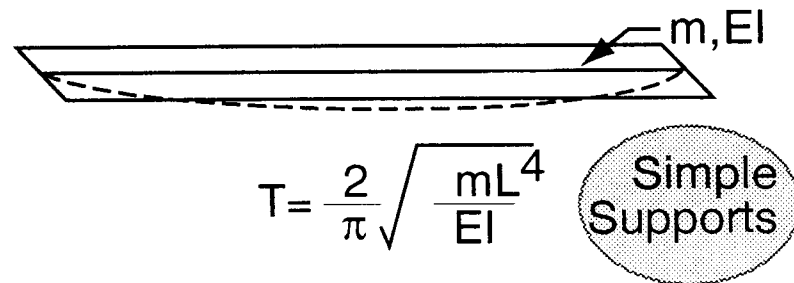


# Check of Periods

- Rigid Body Structure  
(Assume Structure  
Moves as Rigid Body)

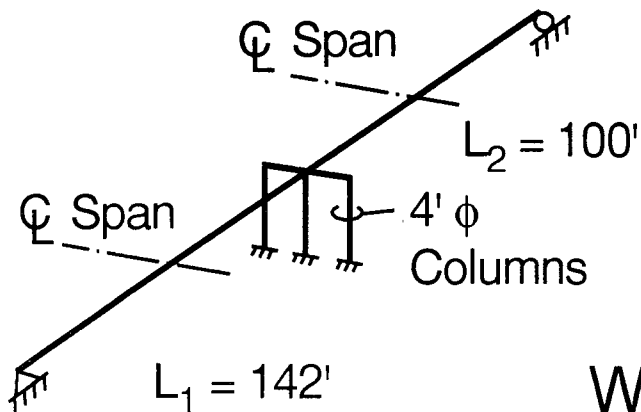


- Use Solution  
for Assumed Shape  
(See Structural Dynamics Texts)



## Example Check – Transverse / Rigid Body

---



$$E_{\text{col}} = 518,420 \text{ ksf}$$

$$I_{\text{col}} = 12.57 \text{ ft}^4$$

$$H_{\text{clr}} = 27.33 \text{ ft}$$

$$W = 2438 \text{ kip}$$

Tributary Weight (1/2 of Each Span)  
Plus 1/2 Column Weight



## Example Check – Transverse / Rigid Body (continued)

---

$$K = 3 \left( \frac{12 E_{\text{col}} I_{\text{col}}}{H^3} \right) = 3 \left( \frac{12 (518,400) 12.57}{(27.33)^3} \right) = 11,500 \text{ kip/ft}$$

$$T = 2\pi \sqrt{\frac{W}{gK}} = 2\pi \sqrt{\frac{2438}{32.2(11,500)}} = 0.510 \text{ sec}$$

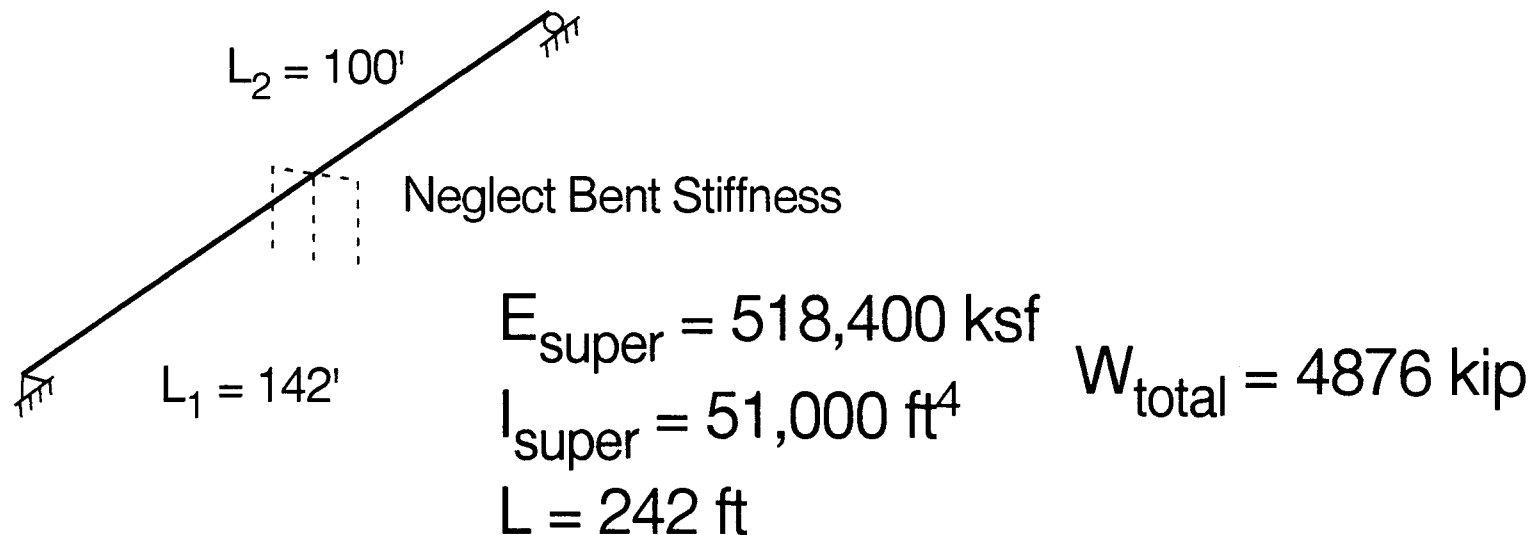
N.G.

Multimode  $T = 0.172 \text{ sec}$

Actual Behavior, More Like Simple Beam

## Example Check – Transverse / Simple Beam Solution

---



## Example Check – Transverse / Simple Beam Solution (continued)

---

Equivalent Distributed Mass,  $m$

$$m = \frac{W_{\text{total}}}{gL} = \frac{4876}{32.2(242)} = 0.626 \frac{\text{ksec}^2}{\text{ft}^2}$$

$$T = \frac{2}{\pi} \sqrt{\frac{mL^4}{EI}} = \frac{2}{\pi} \sqrt{\frac{0.626 (242)^4}{518,400 (51,000)}} = 0.181 \text{ sec}$$

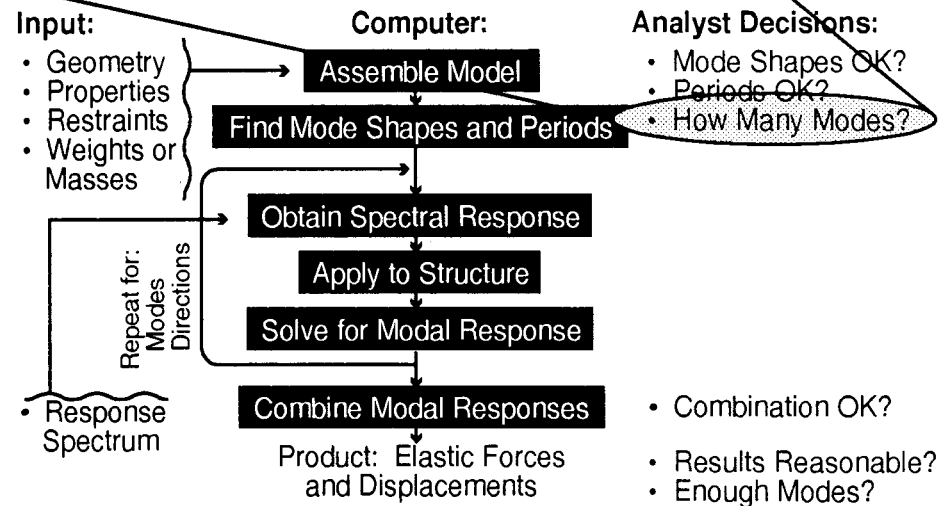
Reference: Structural  
Dynamics Texts

Multimode  $T = 0.172 \text{ sec}$

5% OK

# Multimode Dynamic Analysis

## • How Many Modes?



# Not All Modes Are Required

---

Response Can Be Estimated with Several Modes  
in Each Direction, Typically

- **AASHTO Recommends**

- 3 • No. Spans  $\leq$  25 Modes

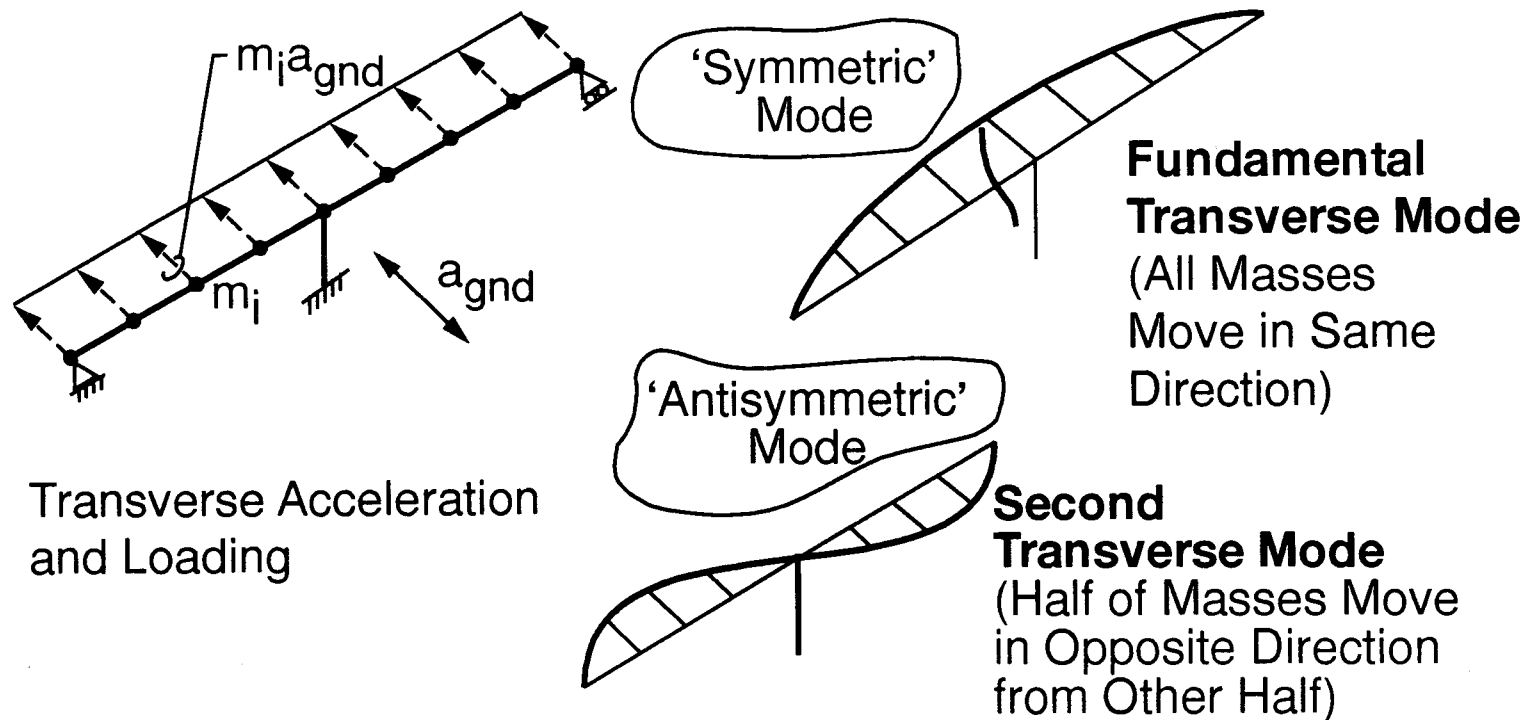
- **Other Recommendations**

- 4 • No. Spans, No Upper Limit

- Participating Mass, 90 – 95%

- Make Sure All Parts of Structure Move

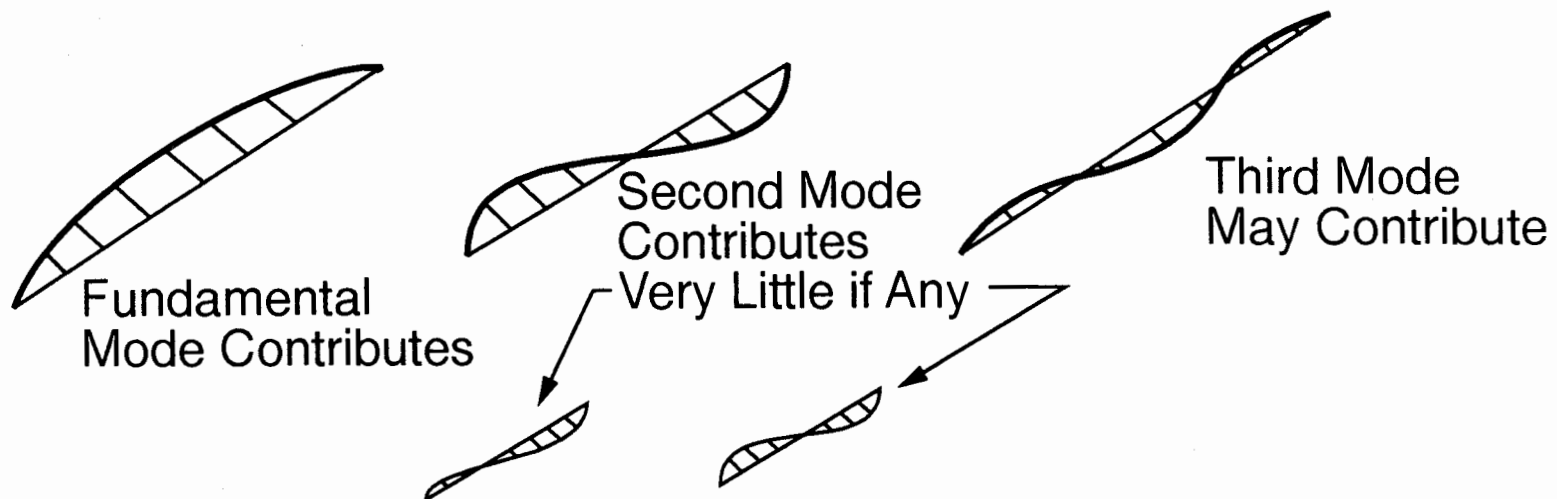
# Modal Participation



## Modal Participation (continued)

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**Result:** Earthquake Loading Will Tend to Excite Only Those Modes that Have a Net Translation in Earthquake Direction



## Rating the Importance of Each Mode

---

**Participating Mass, PM** = Base Shear Contributed  
by Each Mode for a Constant  
Spectral Acceleration

$$PM = \frac{\beta^2}{\gamma} \frac{100}{\text{Total Weight}} \quad (\% \text{ of Structure Weight})$$

Constants — Single-Mode Method Definitions

$\beta$  — Earthquake Excitation for Each Mode

$\gamma$  — Effective Weight for Each Mode



# How Many Modes Are Required?

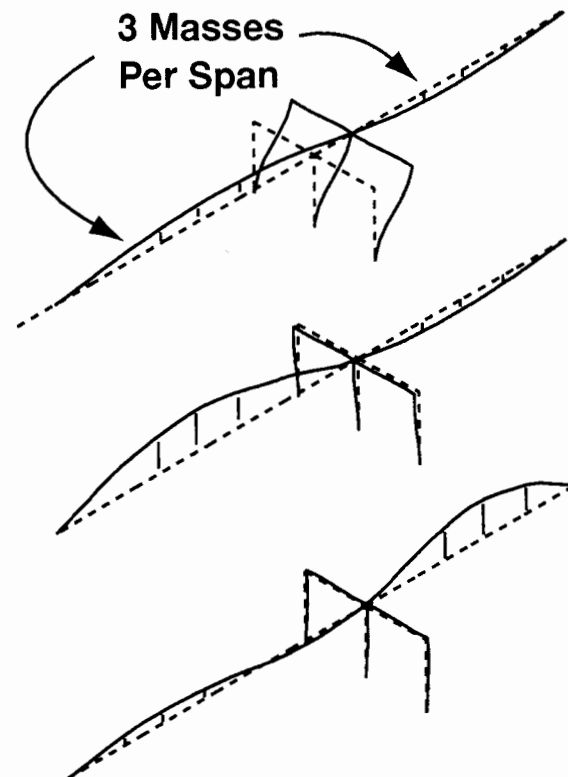
- In Our Example, There Are **38 Modes**, Total
- The First 15 Modes Have Been Determined

PARTICIPATING MASS - (percent)

MODE	X-DIR	Y-DIR	Z-DIR	X-SUM	Y-SUM	Z-SUM	
1	93.148	0.646	0.000	93.148	0.646	0.000	
2	4.807	26.201	0.000	97.955	26.847	0.000	
3	0.322	45.473	0.000	98.277	72.321	0.000	
4	0.000	0.000	88.890	98.277	72.321	88.890	
5	0.032	4.434	0.000	98.309	76.754	88.890	
6	0.013	0.000	0.000	98.322	76.755	88.890	← AASHTO
7	0.000	7.999	0.000	98.322	84.754	88.890	(3x Spans)
8	0.000	0.000	0.000	98.322	84.754	88.890	←
9	0.000	0.000	0.000	98.322	84.754	88.890	4x Spans
10	0.000	12.396	0.000	98.322	97.151	88.890	
11	0.001	1.149	0.000	98.323	98.300	88.890	
12	0.000	0.000	0.000	98.323	98.300	88.890	
13	0.000	0.000	7.103	98.323	98.300	95.993	← 90-95%
14	0.000	0.000	0.000	98.323	98.300	95.993	
15	0.000	0.000	0.000	98.323	98.300	95.993	

# Example Bridge / Participating Mass

Participating Mass (%)			
Mode	Longitudinal	Vertical	Transverse
1	93.2	0.6	0.0
(First Longitudinal Mode)			
2	4.8	26.2	0.0
(First Vertical Mode)			
3	0.3	45.5	0.0
(Second Vertical Mode)			



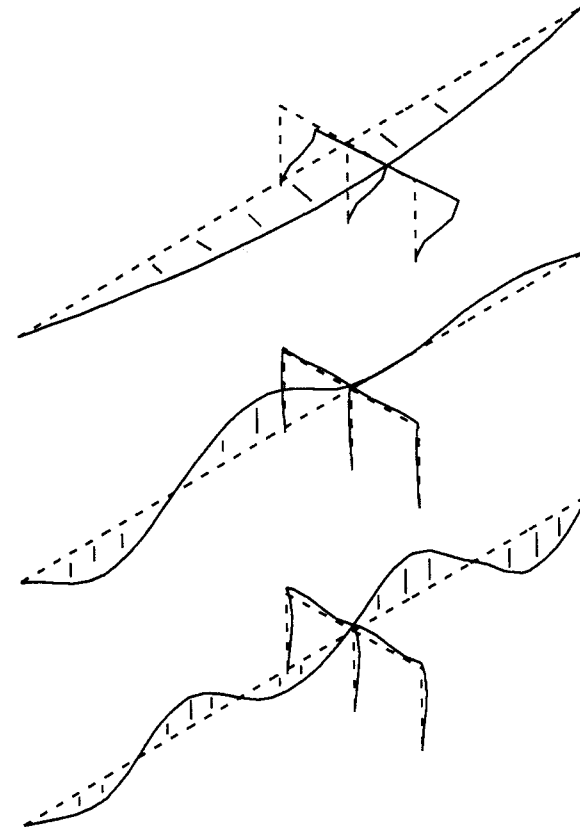
# Example Bridge / Participating Mass (continued)

Participating Mass (%)			
Mode	Longitudinal	Vertical	Transverse

4	0.0	0.0	88.9
(First Transverse Mode)			

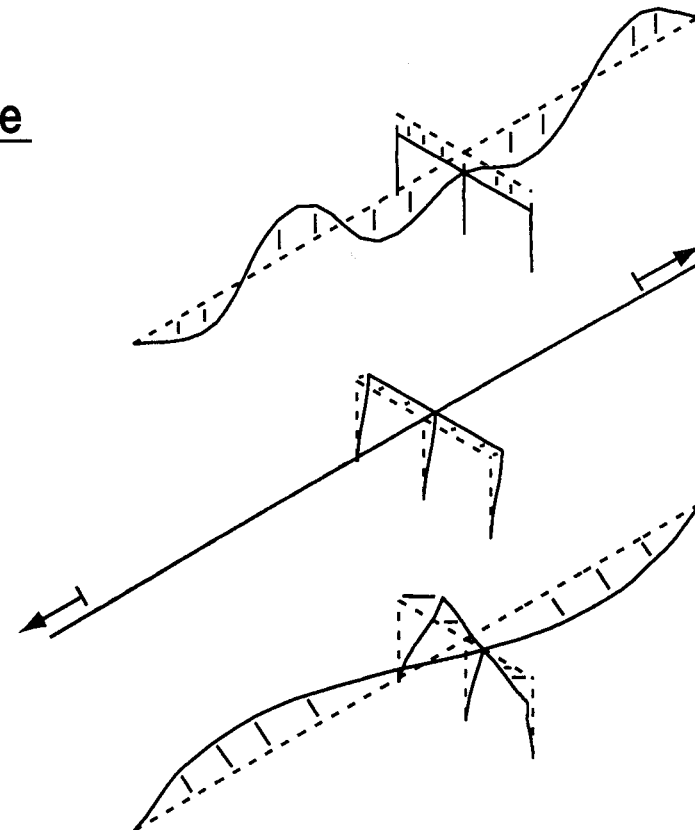
5	0.03	4.4	0.0
---	------	-----	-----

6	0.01	0.0	0.0
---	------	-----	-----



# Example Bridge / Participating Mass (continued)

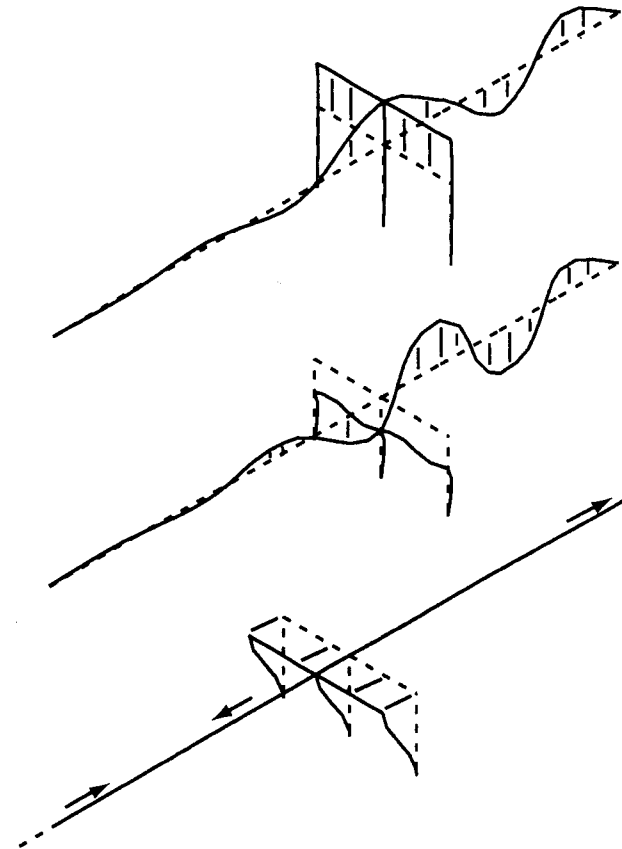
Participating Mass (%)			
Mode	Longitudinal	Vertical	Transverse
7	0.0	8.0	0.0
8	0.0	0.0	0.0
9	0.0	0.0	0.0
(Second Transverse Mode)			



# Example Bridge / Participating Mass (continued)

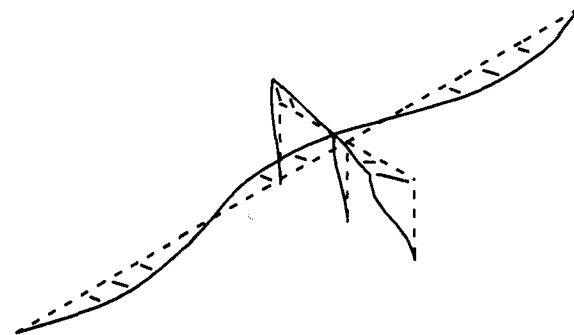
Participating Mass (%)			
Mode	Longitudinal	Vertical	Transverse
10	0.0	12.4	0.0
11	0.0	1.1	0.0
12	0.0	0.0	0.0

(Second Longitudinal Mode)



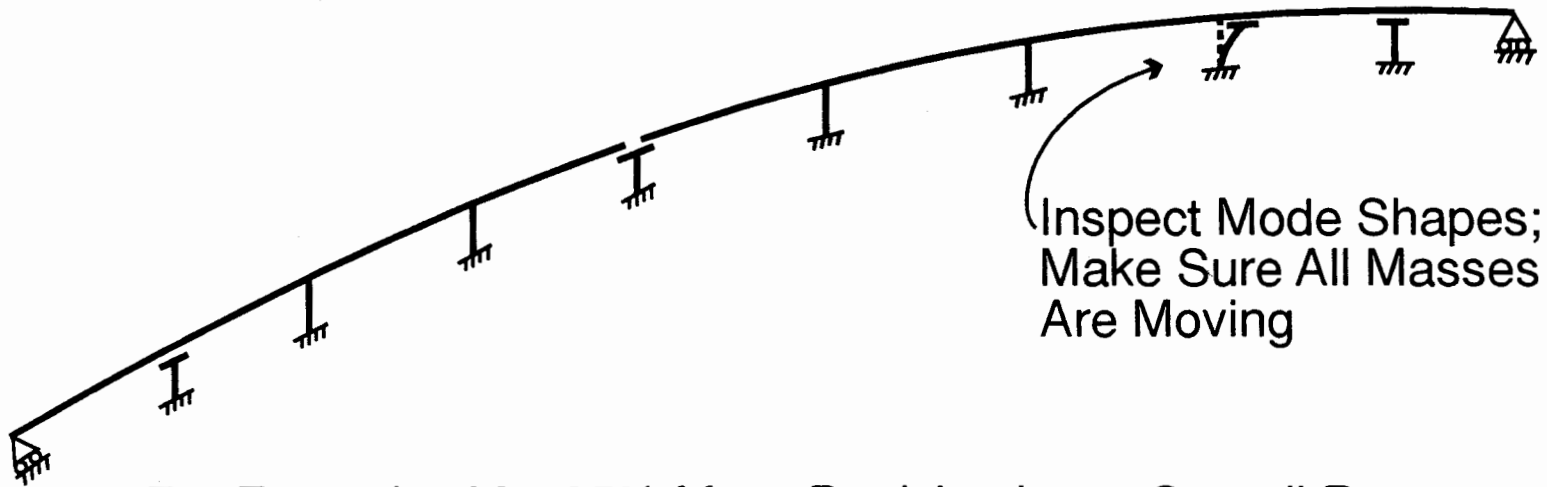
# Example Bridge / Participating Mass (continued)

Participating Mass (%)			
Mode	Longitudinal	Vertical	Transverse
13	0.0	0.0	7.1
(Third Transverse Mode)			
Totals	98.3%	98.3%	96.0%



# Global vs. Local Response Considerations

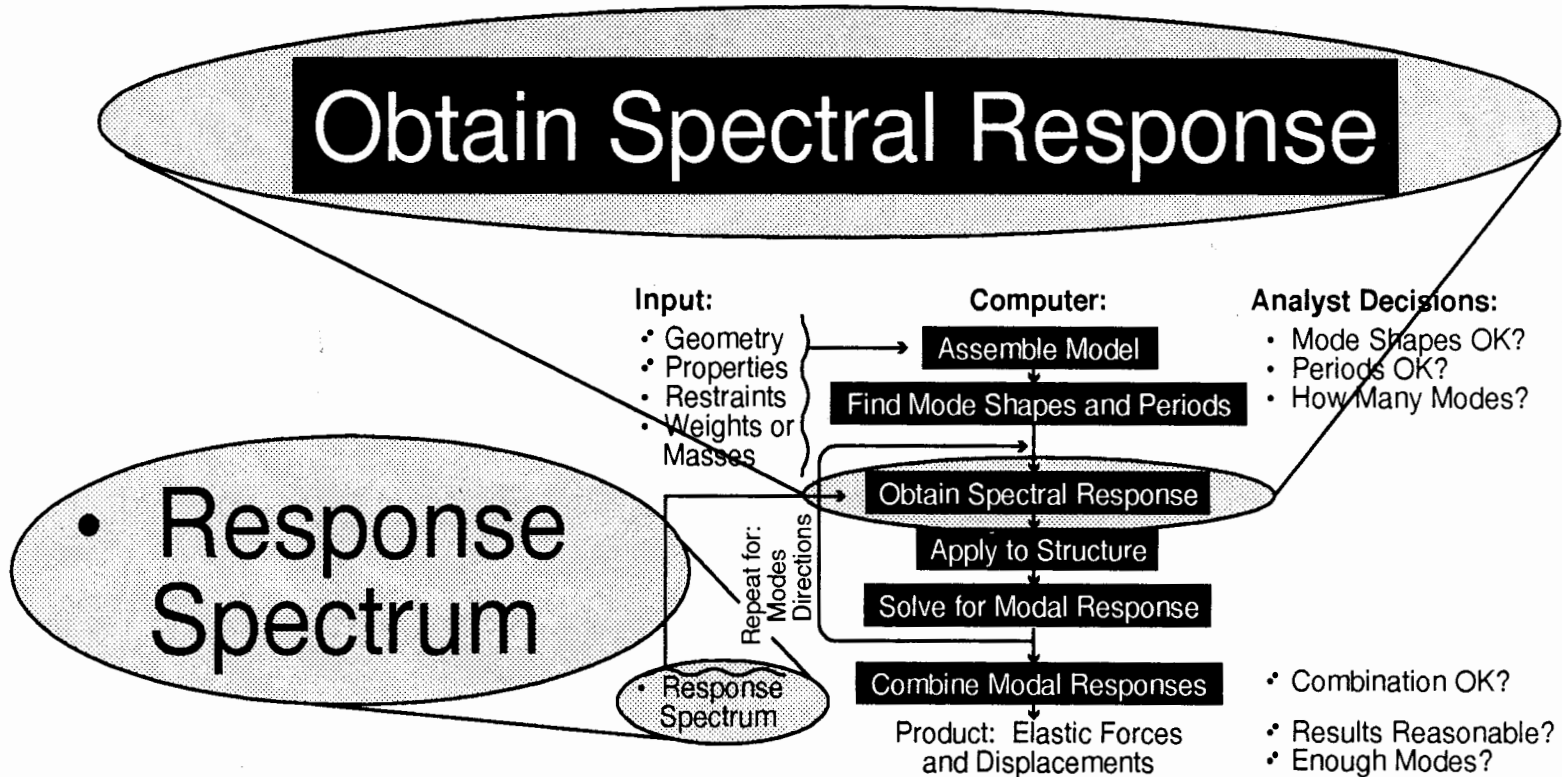
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For Example: 90 - 95% Mass Participation → Overall Response Adequate

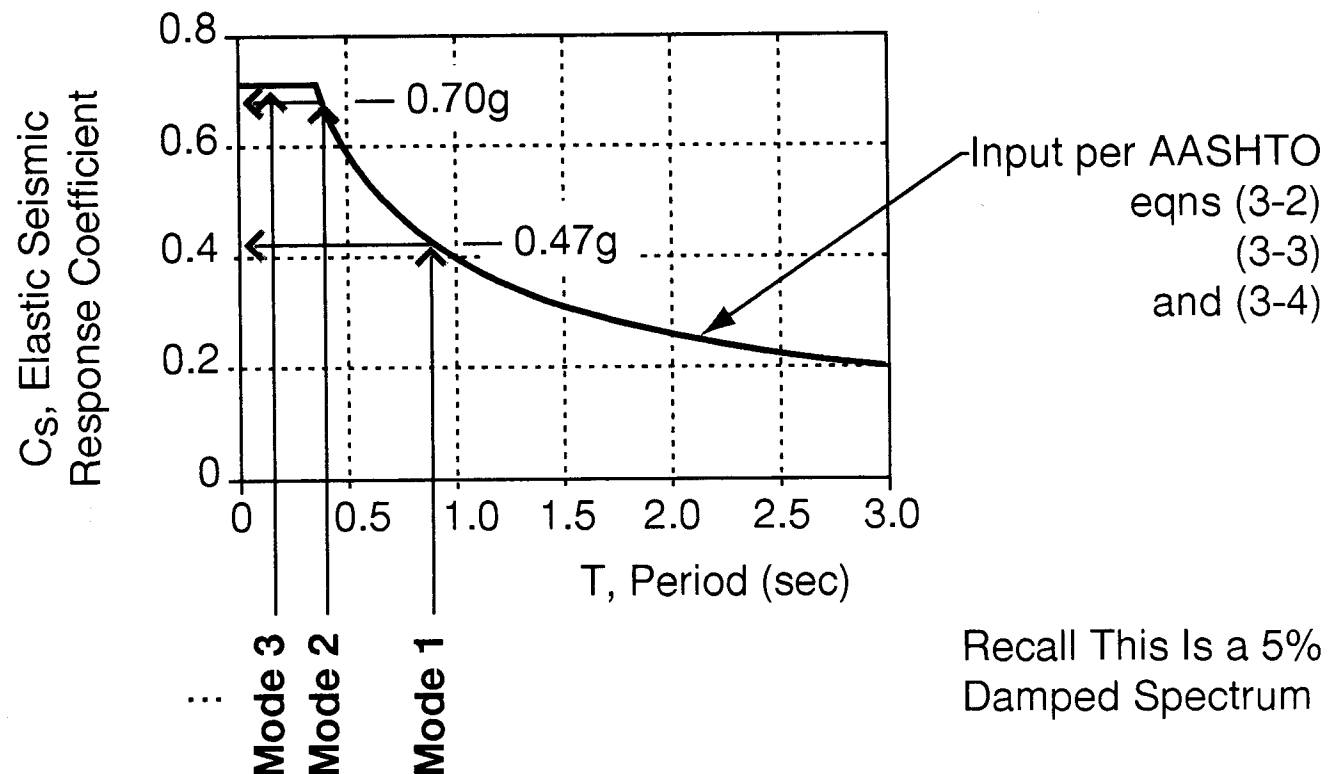
However: Additional Modes May Have Large Impact on Local Response ... Say Forces at a Given Pier

# Multimode Dynamic Analysis

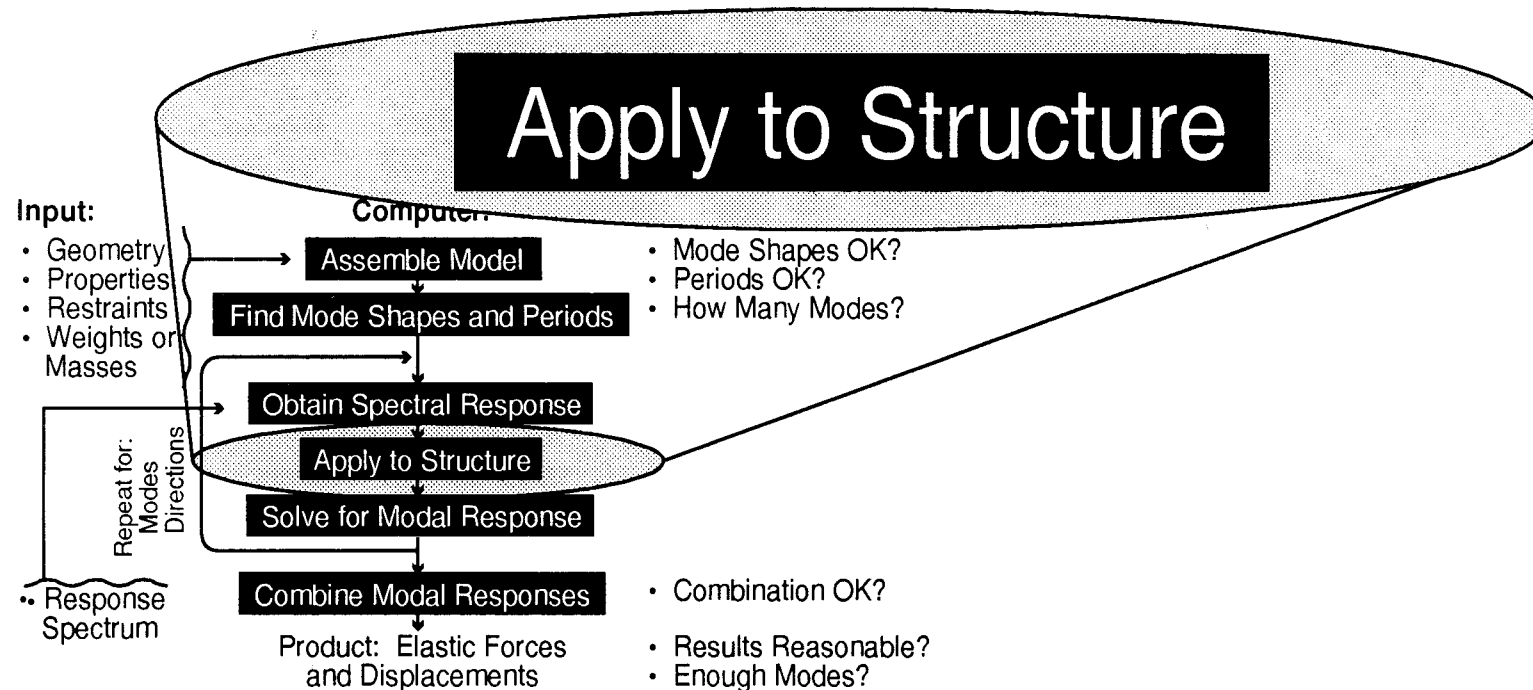




# AASHTO Response Spectrum / Example Bridge



# Multimode Dynamic Analysis



# Weighting Factors for Each Mode

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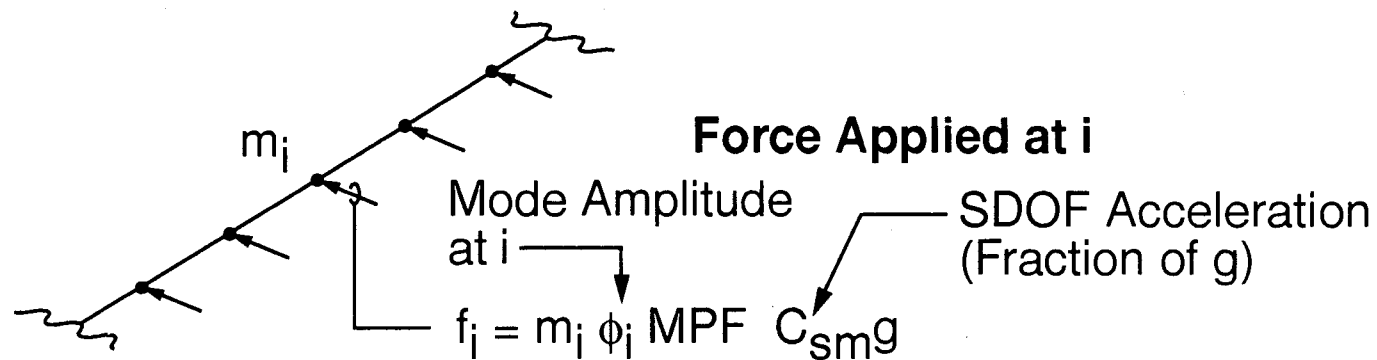
## Modal Participation Factor, MPF

$$\text{MPF} = \frac{\beta}{\gamma}$$

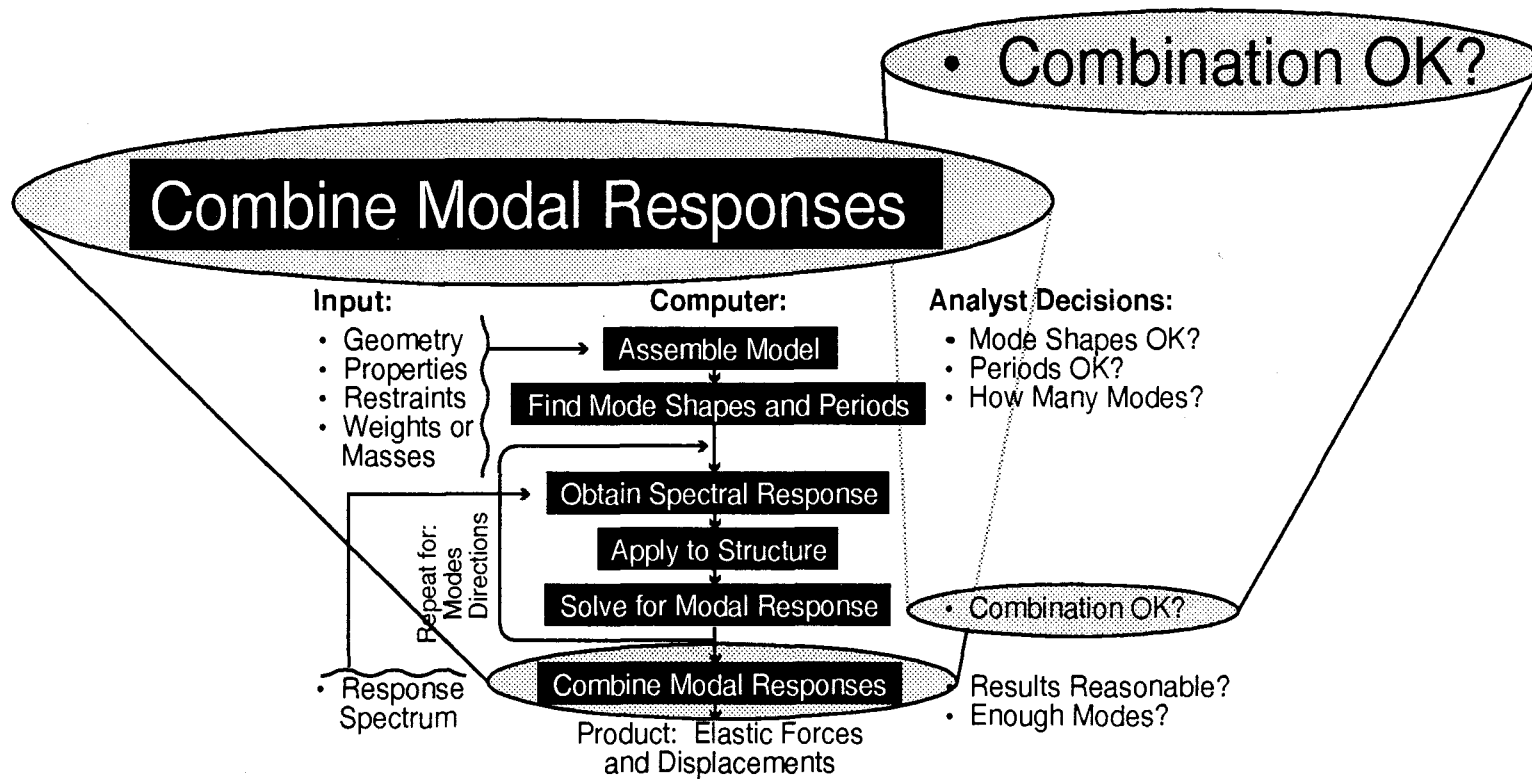
Scales Each Mode's Contribution  
(Analogy: How Many Parts of Color for Paint?)

↑  
Same as Single-Mode Factors

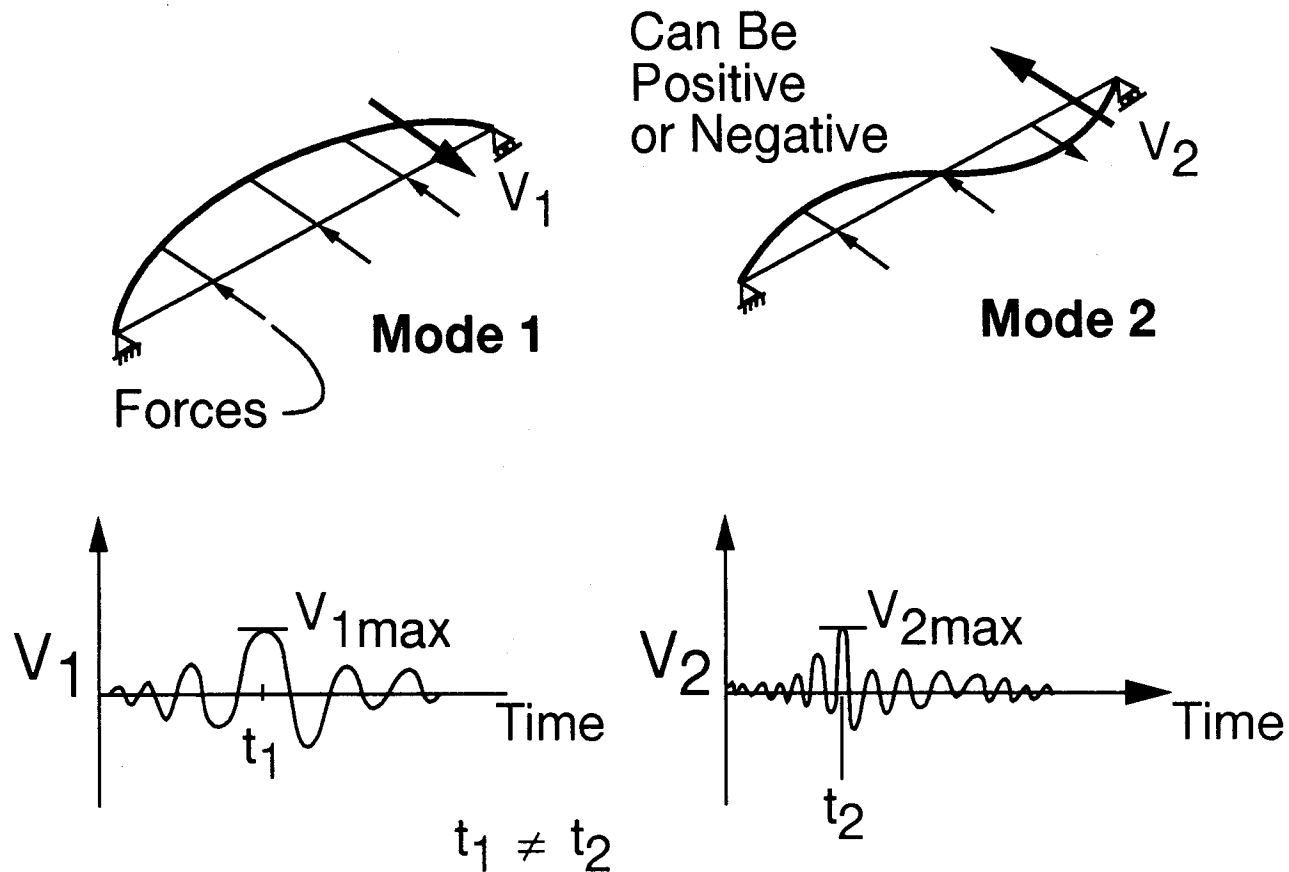
# Apply Forces to Structure



# Multimode Dynamic Analysis



# Combining Modal Forces



# Combining Modal Responses

---

- Since Maxima Do Not Occur at the Same Time, Adding Results May Be too Conservative

**'SAV'** — Sum of Absolute Values, too Big

- Could Use

**'SRSS'** — Square Root of Sum of the Squares

$$V = \sqrt{V_1^2 + V_2^2 + \dots}$$

OK, if Periods Are Well Separated

# Combining Modal Responses (continued)

---

- Recommend

**‘CQC’** — Complete Quadratic Combination

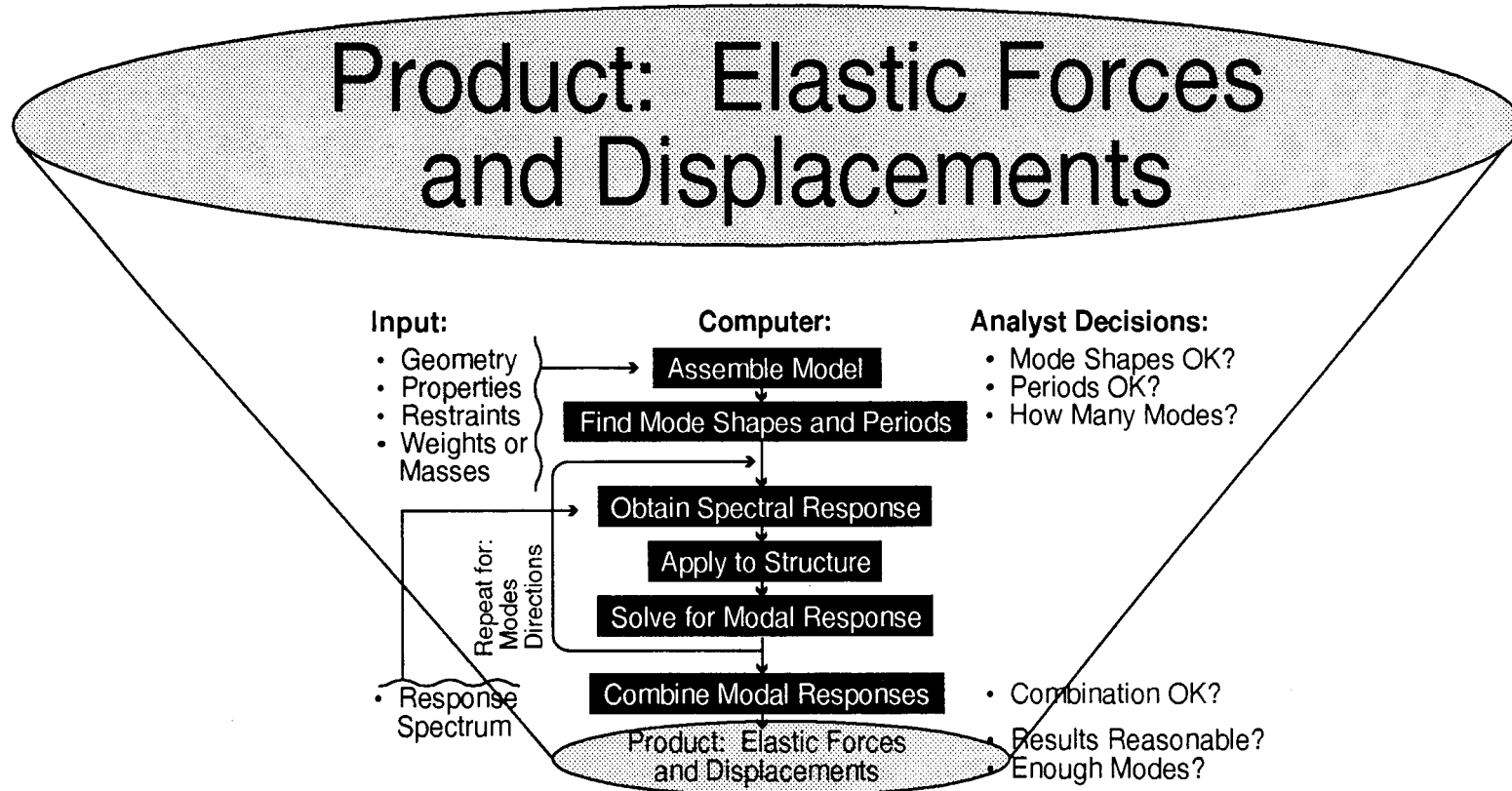
Handles Interaction of Modal Response when  
Periods Are Close

CQC Turns into SRSS for Well-Spaced Modes

↖ ‘Square Root of Sum of the Squares’

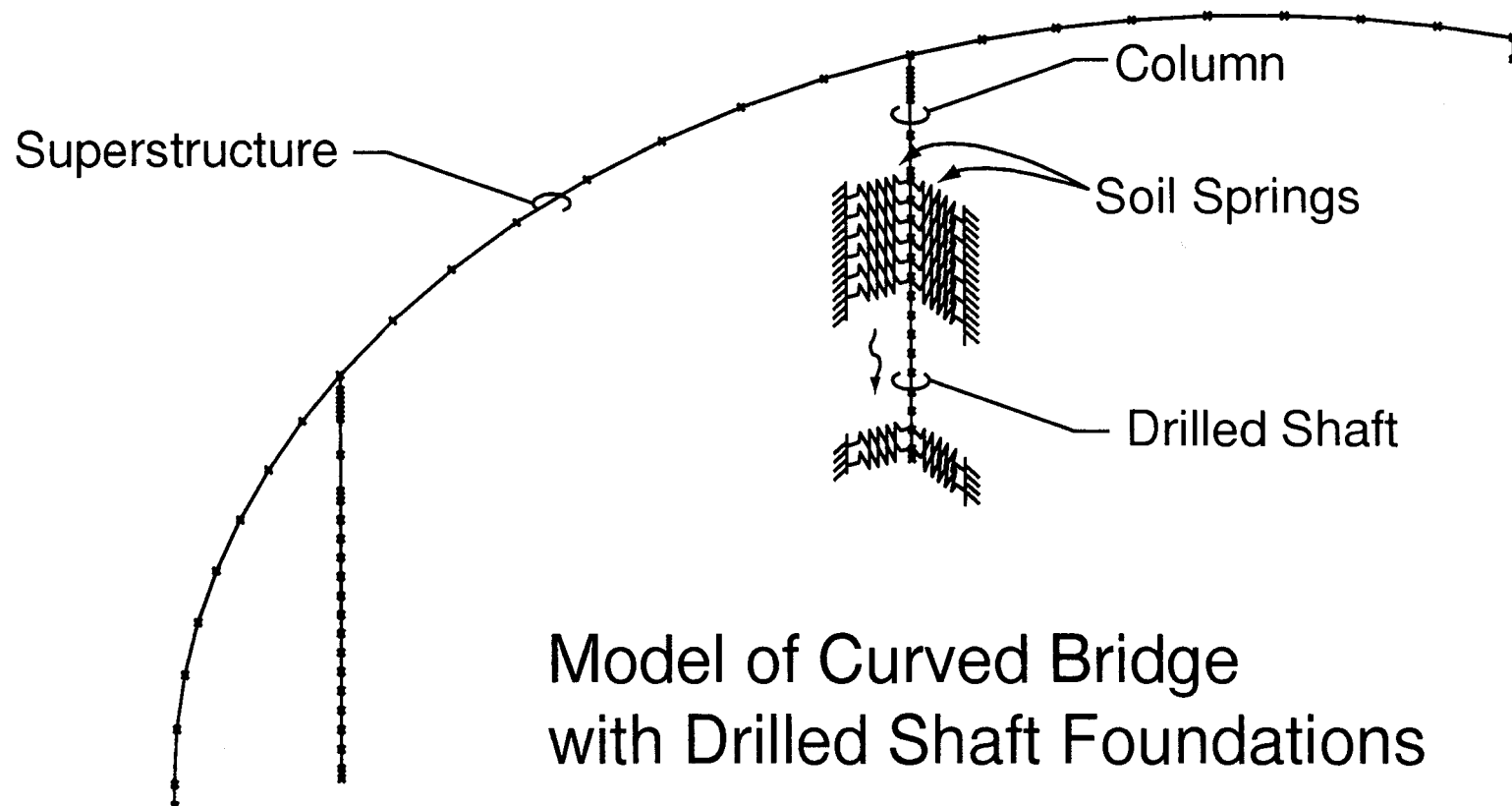


# Multimode Dynamic Analysis

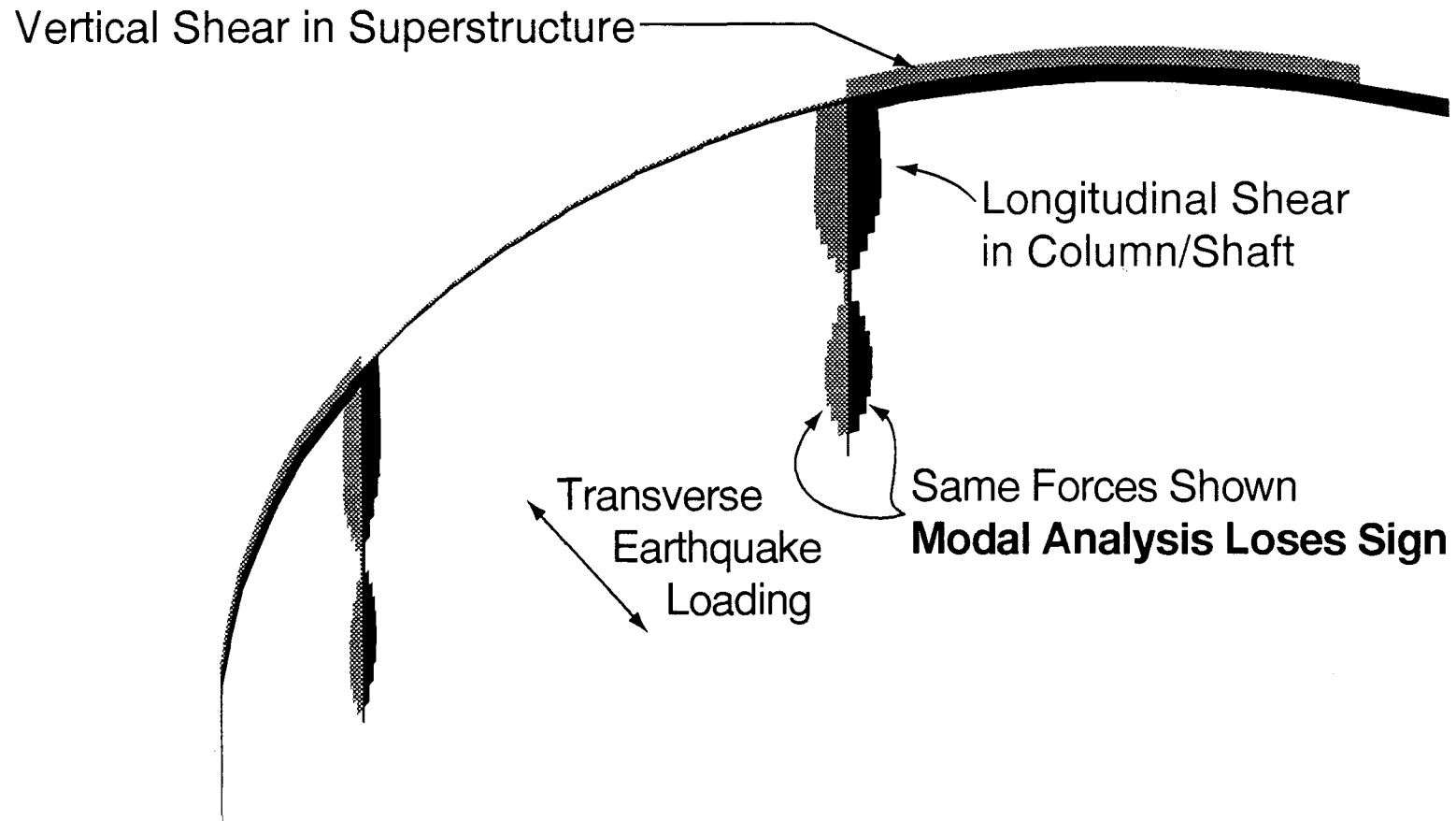


# More Complex Example / Curved Bridge

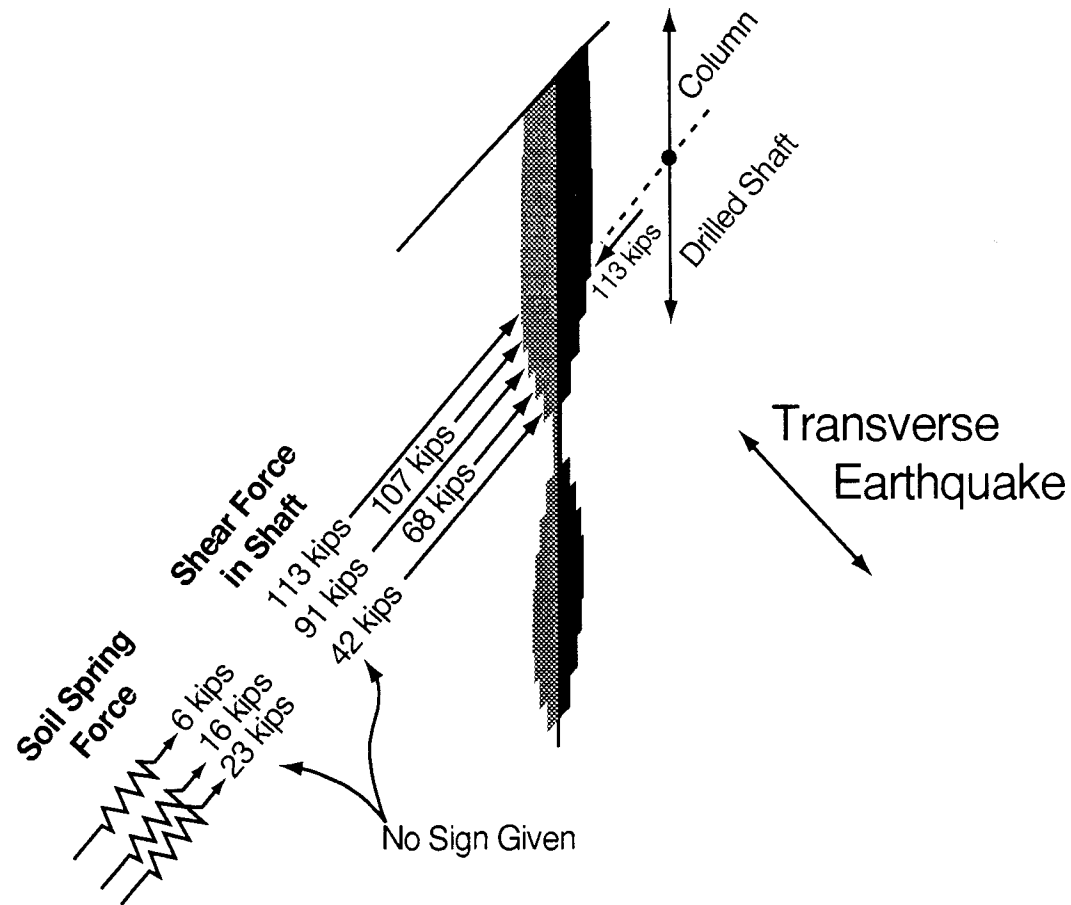
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# Multimode Shear Forces

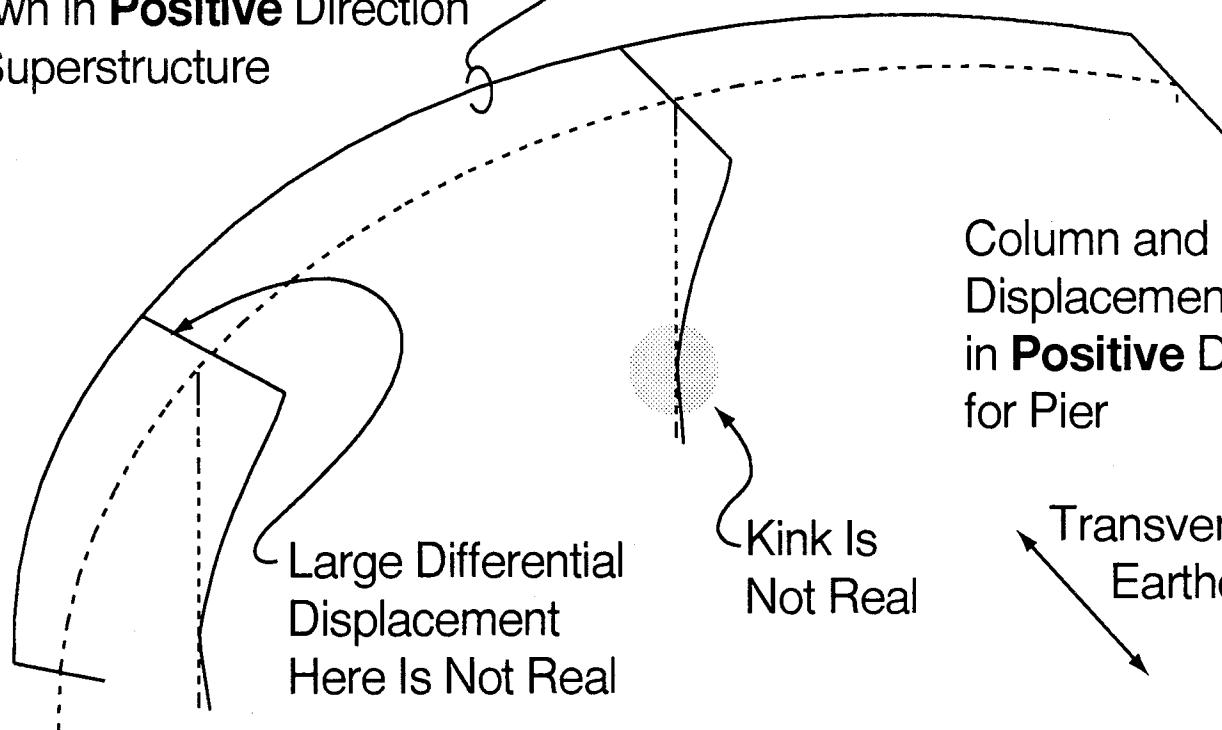


# Zoom in on Column / Shaft



# Multimode Displacements

Superstructure Displacements  
Shown in **Positive** Direction  
for Superstructure



Column and Shaft  
Displacements Shown  
in **Positive** Direction  
for Pier

Transverse  
Earthquake

## Interpretation of Results

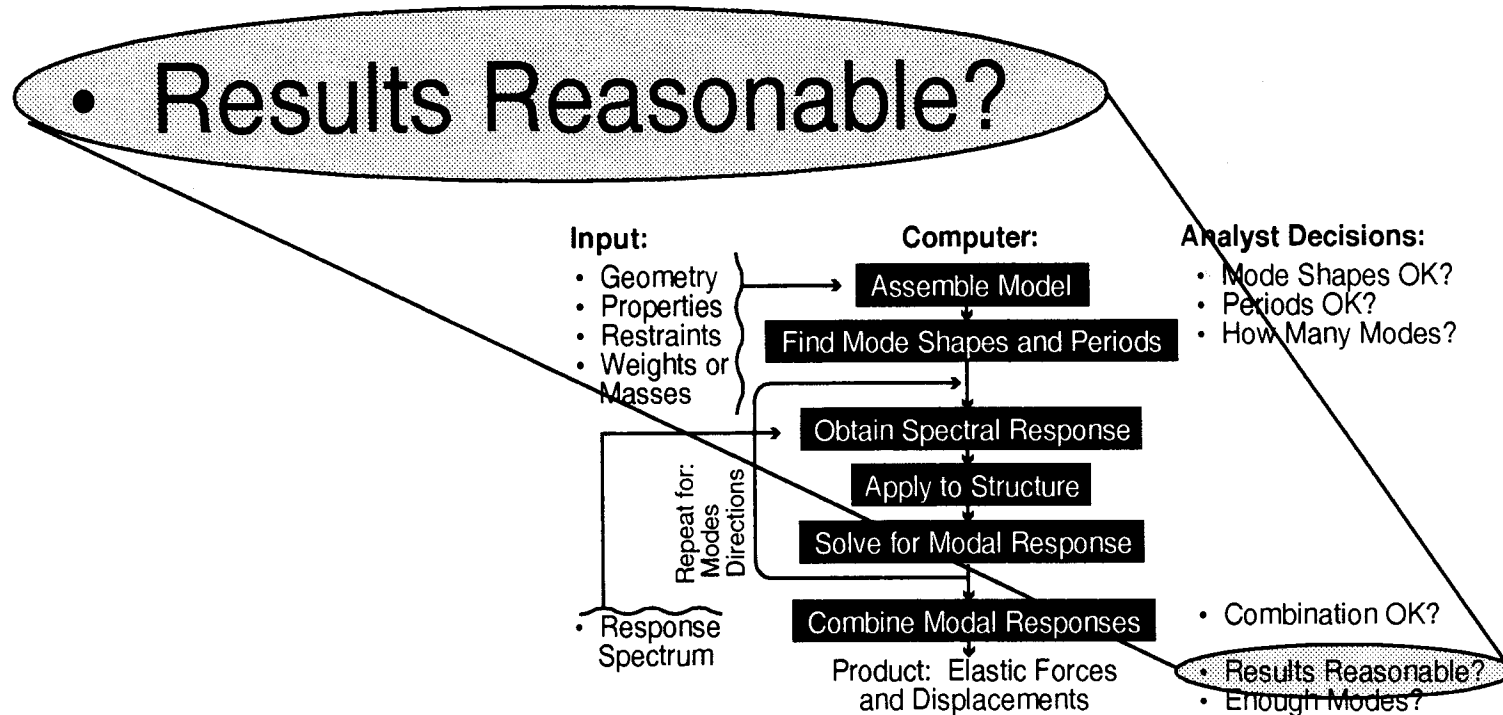
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- Forces, Reactions, and Displacements Are Reported as Positive

$$\left( \sqrt{F^2} \right)$$

- Due to Loss of Sign, Equilibrium Checks Are Difficult or Impossible
- Statics Checks Are Possible on a Mode-by-Mode Basis (i.e., Each Mode Separately)

# Multimode Dynamic Analysis



## Example – Check Total Transverse Shear

**Could Use:**

$$V = C_s W \quad W = 4876 \text{ kip}$$

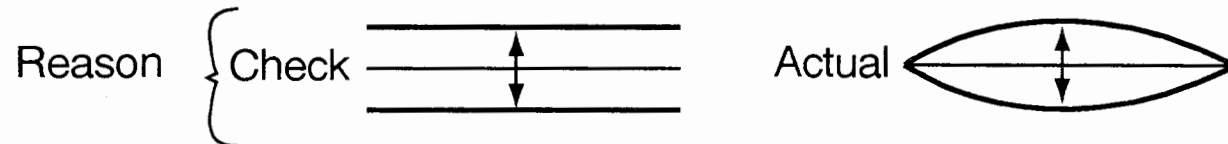
$$C_s = \frac{1.2 AS}{T^{2/3}} = \frac{1.2 (0.28) 1.2}{(0.181)^{2/3}} = 1.26 \leq 2.5 A = 2.5(0.28) = 0.70$$

Controls

$$V = 0.70(4876) = 3413 \text{ kip Total Shear}$$

$$V_{\text{multimode}} = 2735 \text{ kip}$$

20% High





## Example – Check Total Shear (continued)

Or Use Simple  
Beam Solution:

$$V_s(x) = \sin \frac{\pi x}{L}$$

$$V = PM \cdot C_s$$

$$\beta = \int_0^L w \sin \frac{\pi x}{L} dx = \frac{wL}{\pi}$$

$$\gamma = \int_0^L w \sin^2 \frac{\pi x}{L} dx = \frac{wL}{2}$$

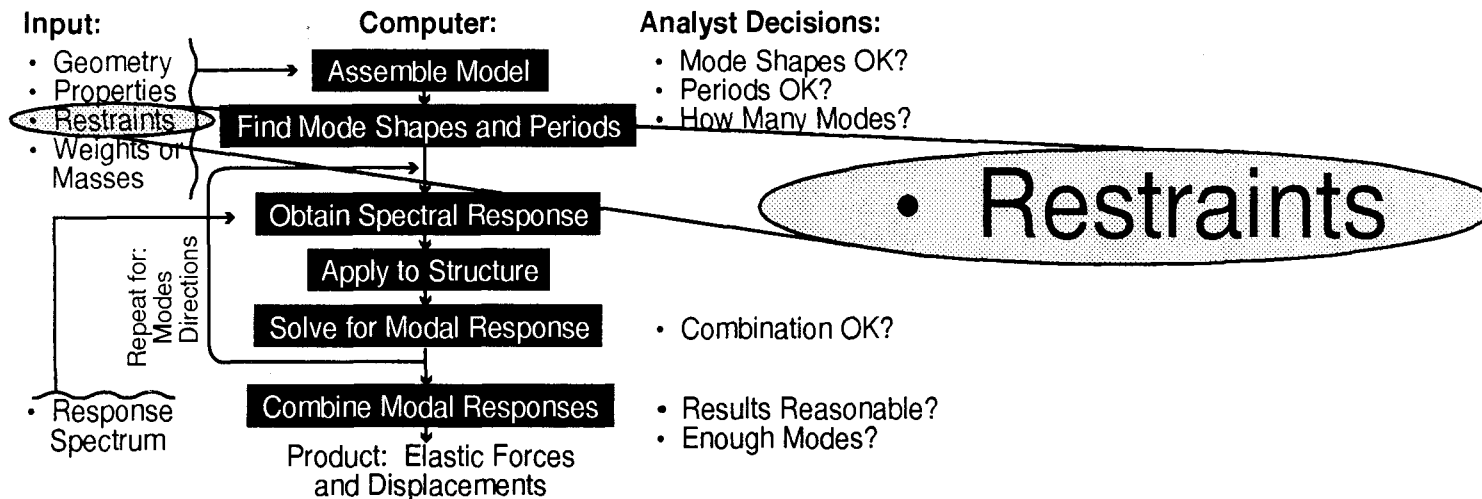
$$V = \frac{\beta^2}{\gamma} \cdot C_s = \frac{w^2 L^2}{\pi^2} \frac{2}{wL} \cdot C_s = wL \frac{8}{\pi^2} \cdot C_s = W(0.811)C_s$$

$$V = 4876 (0.811) 0.70 = 2767 \text{ kip}$$

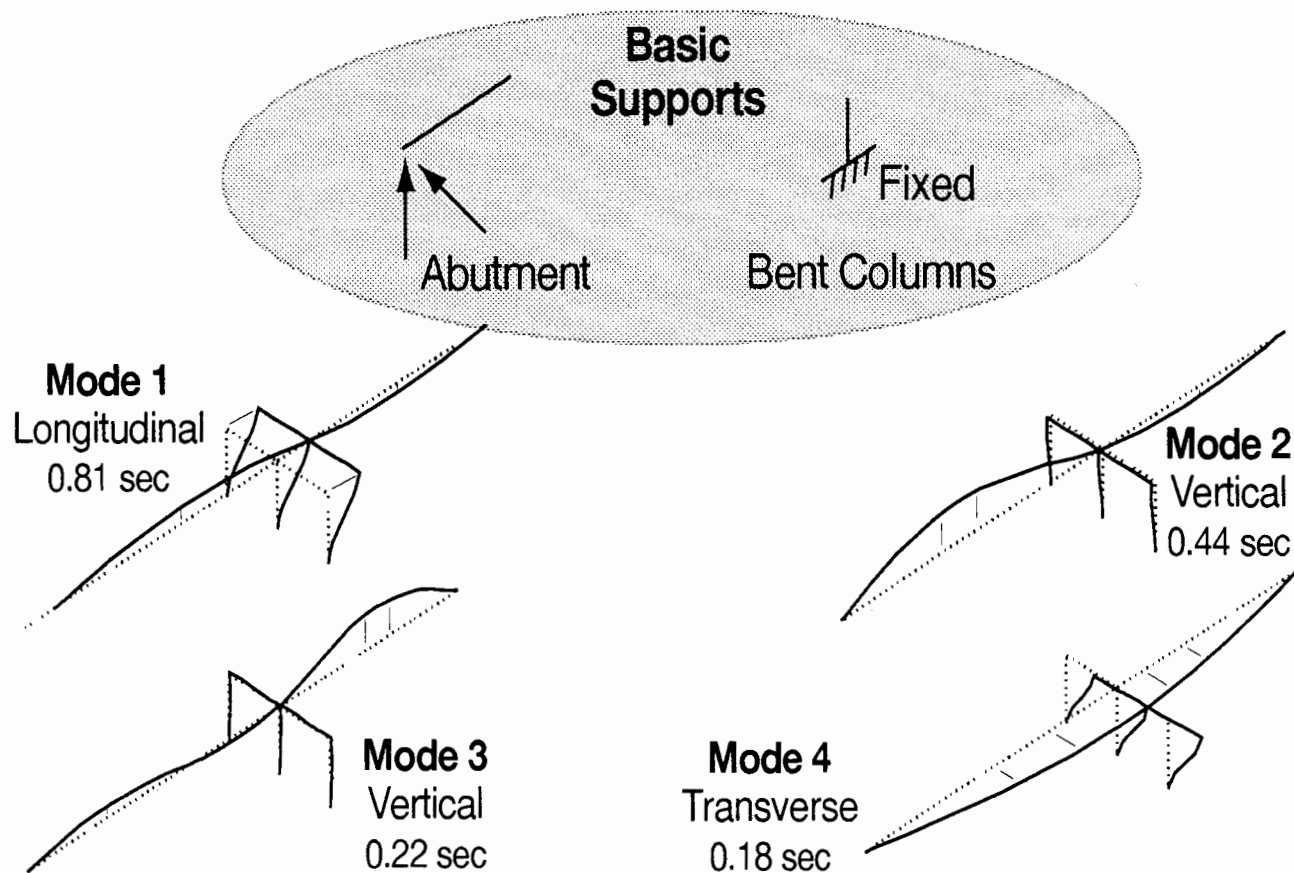
$$V_{\text{multimode}} = 2735 \text{ kip}$$

1% Difference ✓

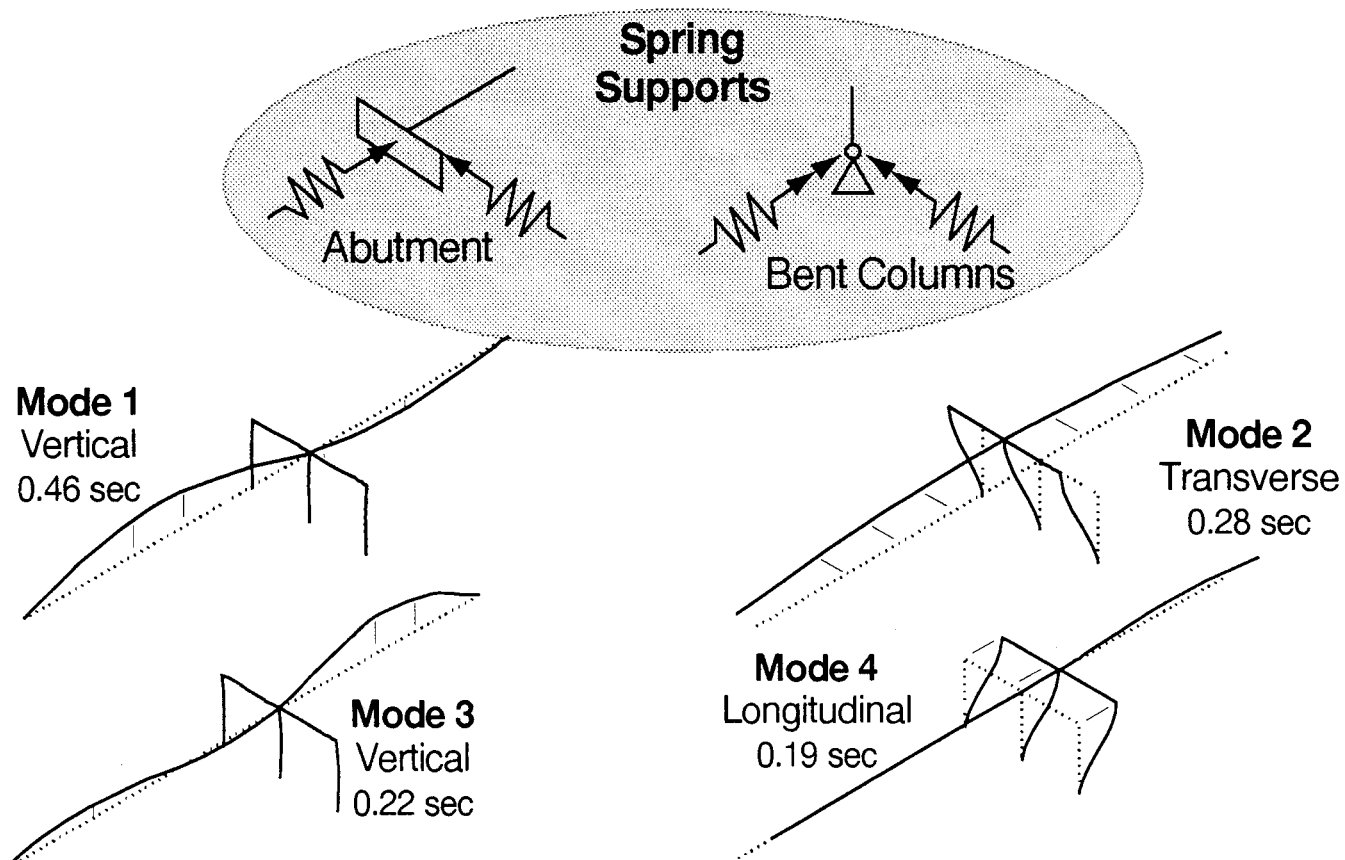
# Multimode Dynamic Analysis



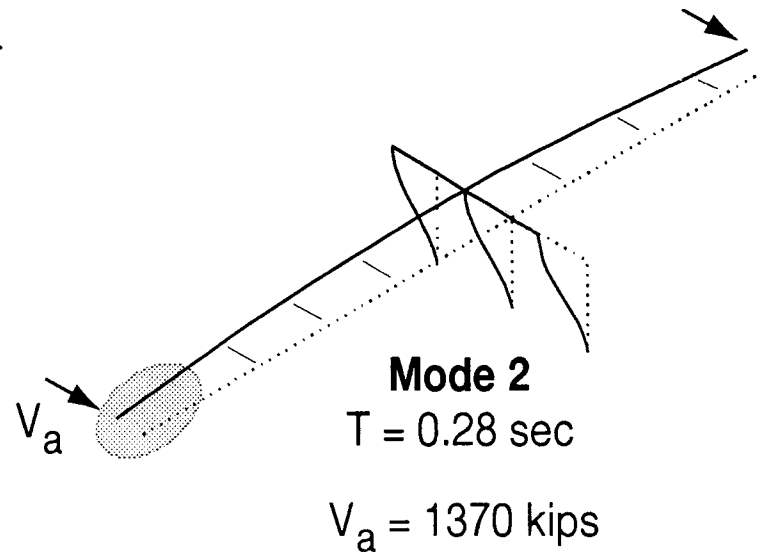
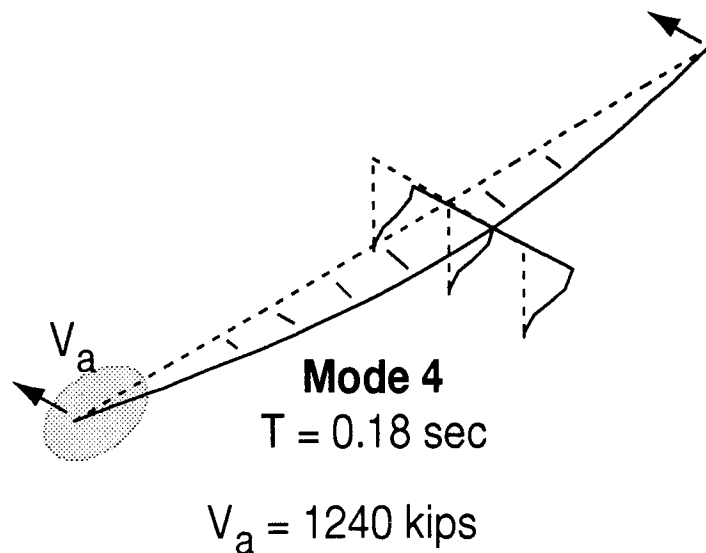
# Effects of Support Conditions on Mode Shapes



# Effects of Support Conditions on Mode Shapes



# Effects of Support Conditions on Forces



- Shear May Increase with Springs Even though Period Is Longer
- **Reason:** Superstructure Is Moving More, which Increases Inertial Forces



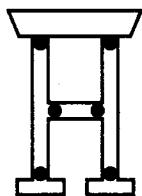
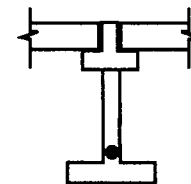
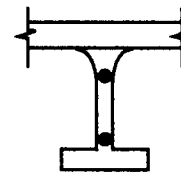
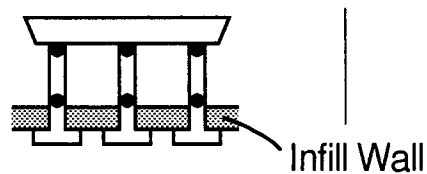
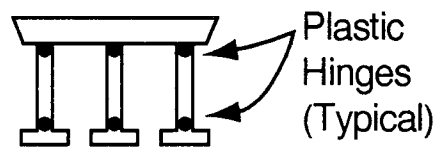
# **Session 7**

## **Column and Pier Design**

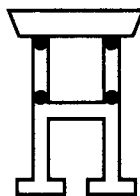
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- **Intended Seismic Behavior**
- **SPC B vs. SPC C and D Requirements**
- **Wall Pier Design**
- **General Detailing Issues**

# Plastic Hinging Locations / Mechanism



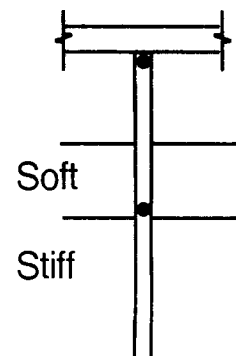
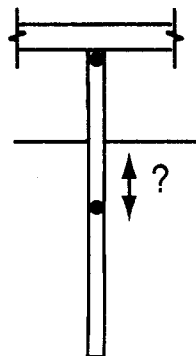
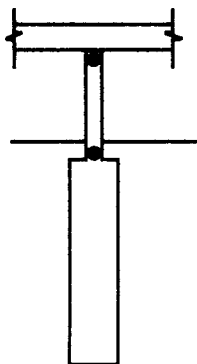
or



...

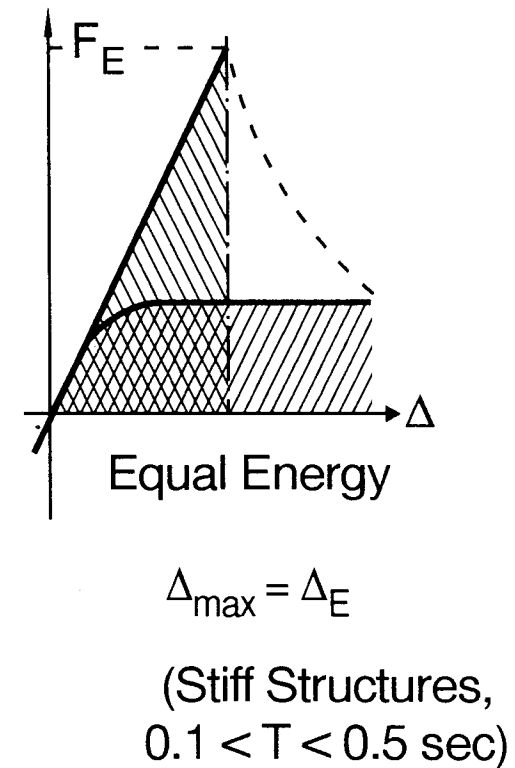
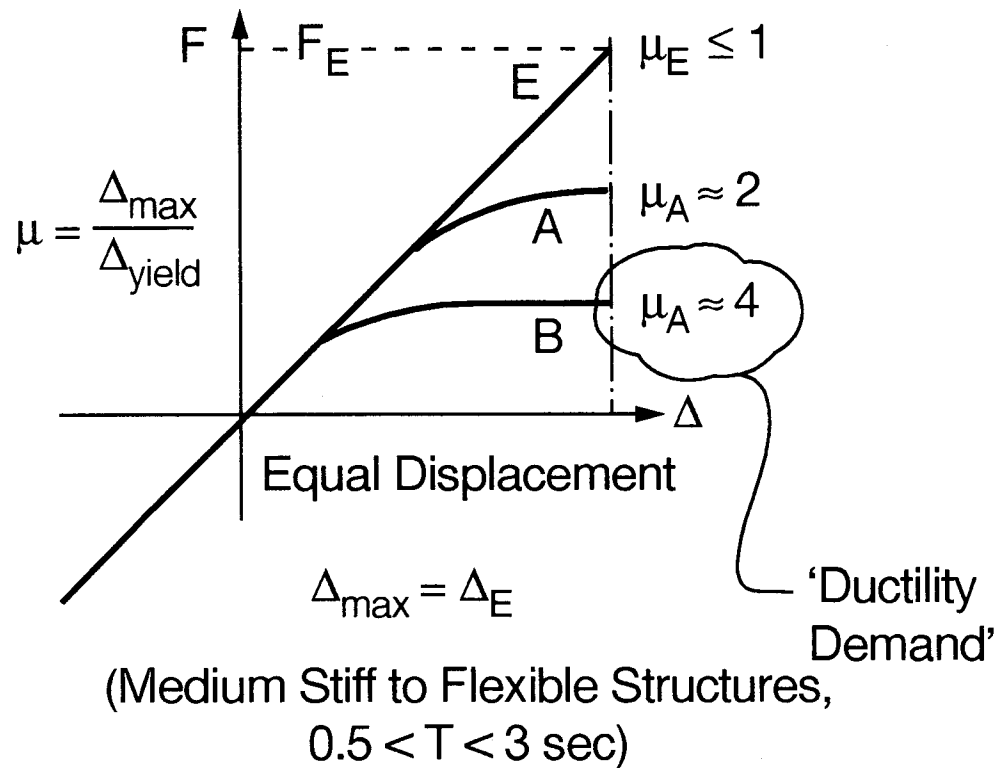
Trial and Error

'Upper Bound Theorem' — The Lateral Force for an Assumed Mechanism Is Greater than or Equal to the True Force





# Consequences of Allowing Yielding in Structures



# Consequences of Allowing Yielding in Structures

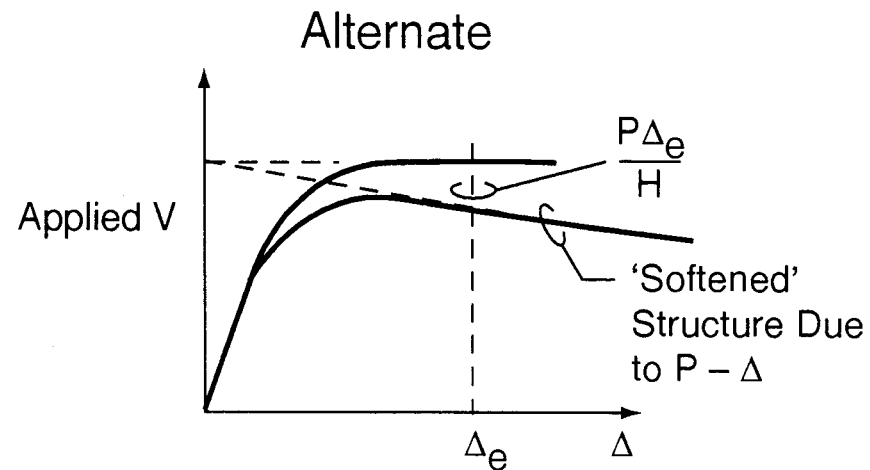
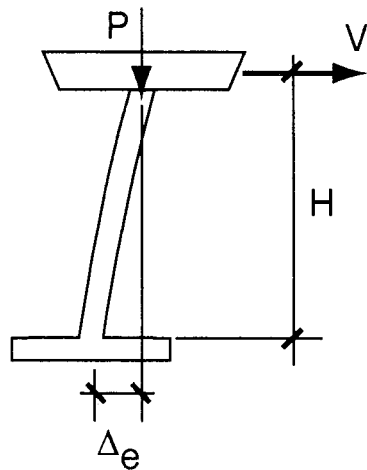
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## Implications:

- Ductility Demand  $\mu$  Increases as Resistance  $F$  Decreases
- As  $\mu$  Increases, Chance of Damage Increases
- As  $\mu$  Increases, Special Detailing Becomes Necessary

# Slenderness and P - $\Delta$ Effects

- AASHTO — Use Division I Method (Elastic Theory)
- Alternate — Increase Design Moment by  $P \cdot \Delta_e$  to Account for Loss of Resistance at  $\Delta_e$  (Concrete)

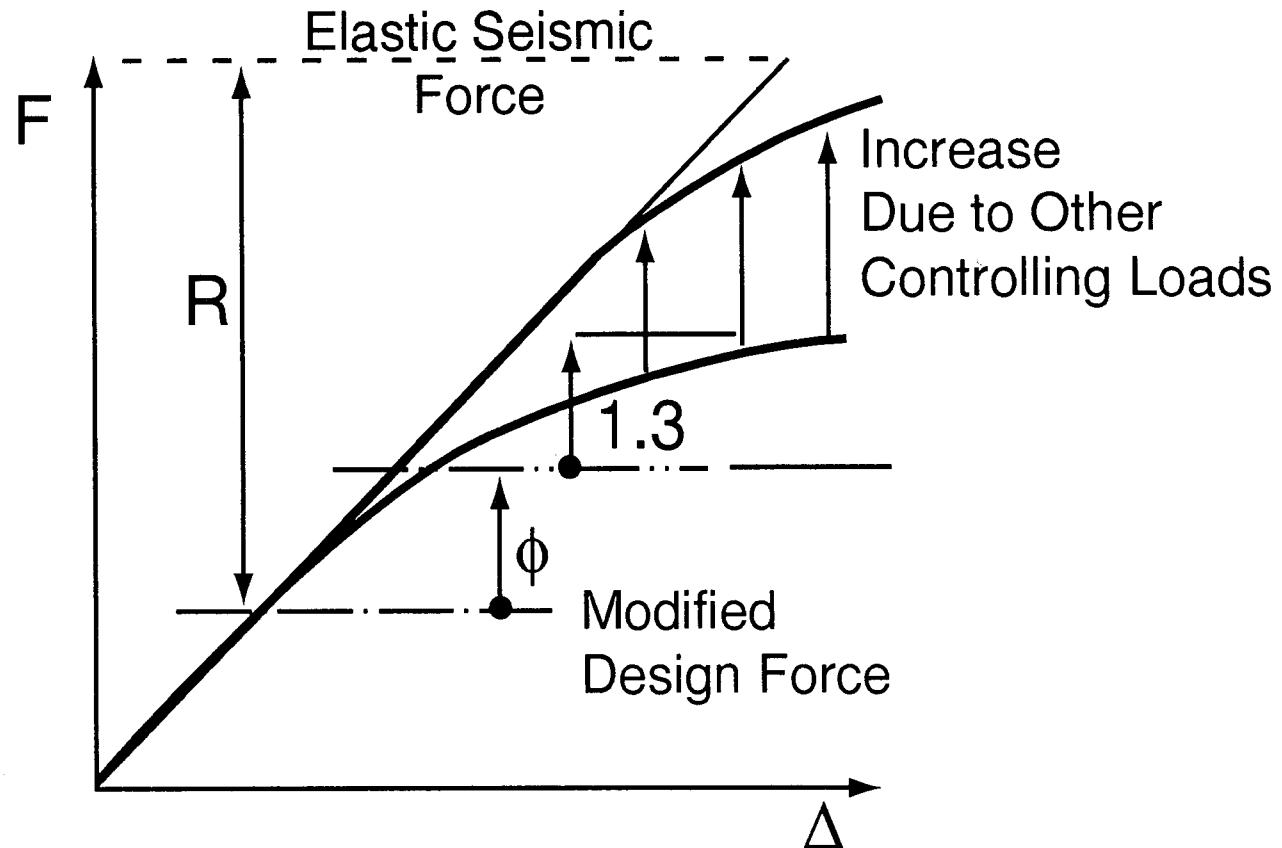


# Non-Seismic Controlling Load Cases

---

- More Common in Lower Acceleration Zones
- Reduces Ductility Demands for Design Ground Motion
- May Significantly Increase Plastic Hinging Shear
- Foundation Sizes May Be Quite Large for Plastic Forces

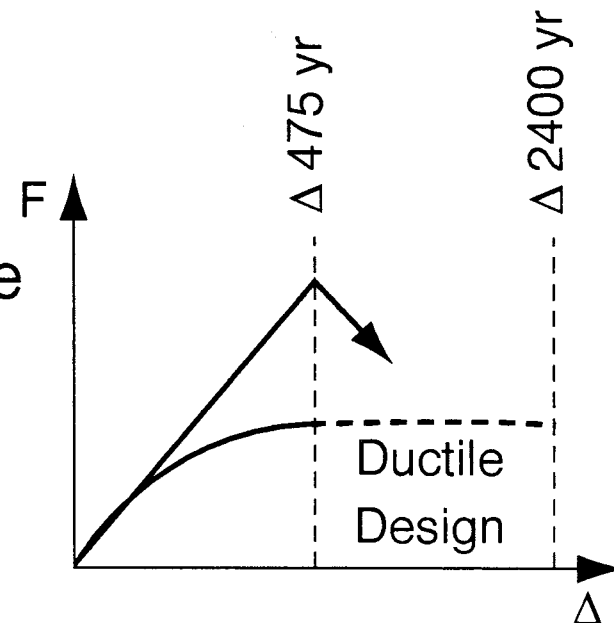
# Recall F vs. $\Delta$ Behavior



# Fail-Safe Issues

- Recall that Design Shaking  
= 10% Chance of Exceedence in 50 Years  
 $\neq$  10% Chance of Being Equal to Acceleration Level

- Provide for Ductile Response  
Up to and **Beyond**  
Design Ground Motion



# Fail-Safe Issues (continued)

Spectral Acceleration for  $T = 0.3$  sec  
(Not Peak Ground Acceleration)

Ground Motion	San Francisco California	Boston Massachusetts
475 Year Return Period (10% Chance of Exceedence in 50 Years)	1.75 g	0.37 g
	1.7x	2.4x
2400 Year Return Period (2% Chance of Exceedence in 50 Years)	3.00 g	0.88 g

# **Session 7**


## **Column and Pier Design**

---

- **Intended Seismic Behavior**
- **SPC B vs. SPC C and D Requirements**
- **Wall Pier Design**
- **General Detailing Issues**



# Force Requirements; SPC B vs. Higher Categories

Design Forces	SPC B	SPC C and D
Column Flexure	$\frac{\text{Elastic}}{R}$	$\frac{\text{Elastic}}{R}$
Column Shear and Axial, Connections	$\frac{\text{Elastic}}{R}$	Plastic Hinging Forces, or Full Elastic Forces (Seismic)
Foundations	$\frac{\text{Elastic}}{R/2}$  Attempts to Force Column to Yield	Plastic Hinging Forces, or Full Elastic Forces (Seismic)

# SPC B – Column Design

---

Consider Our Example Bridge /  $A = 0.15 g$  3 ft Diameter Column

**Column  
Design  
Forces:**

$$\left. \begin{aligned} M_u &= 891 \text{ kip ft} \\ P_u &= 1049 \text{ kip} \\ V_u &= 58 \text{ kip} \end{aligned} \right\}$$

All Based  
on  $R = 5$

8 #10 (1.00%)

$\phi = 0.7$  for SPC B (Instead of  
 $\phi = 0.5$  SPC C and D)

**Plastic  
Hinging  
Forces:**

$$\left. \begin{aligned} M_p &= 1794 \text{ kip ft} \\ V_p &= 142 \text{ kip} \end{aligned} \right\}$$

Not Required  
in SPC B

## SPC B – Column Design (continued)

---

### Implications:

Flexure       $M_p = 1794 \text{ kip ft} < M_{LC1 + DL}^{\text{elastic}} = 3981 \text{ kip ft} \therefore \text{Column Yields}$

Shear       $V_p = 142 \text{ kip} > V_u = 58 \text{ kip} \therefore \text{Problem?}$

$\phi V_n = 135 \text{ kip with Minimum Steel} \therefore \text{Close, but ...}$

Is It Wise to Divide Column Shear by R in SPC B?

# SPC B – Footing Design

---

## Footings Design Forces:

$$\left. \begin{array}{l} M_u = 1497 \text{ kip ft} \\ V_u = 116 \text{ kip} \end{array} \right\} \begin{array}{l} \text{All Based} \\ \text{on } R = \frac{5}{2} = 2.5 \end{array}$$

## Implications:

Rocking       $M_p = 1794 \text{ kip ft} > M_u = 1497 \text{ kip ft}$        $\therefore$  Greater than 1/2 Uplift ?  
(Transferred from Column)      Footing Shear Problems ?

Sliding       $V_p = 142 \text{ kip} > V_u = 116 \text{ kip}$       Sliding Possible ?

# General Statements – SPC B Design

---

**Column Shear:** Actual Shear Capacities May Be Much Less than Plastic Shear Demand

∴ Use  $R = 1$  for Shear  
or Use Procedure for SPC C and D

**Footing Forces:** Use of  $\frac{R}{2}$  Works Reasonably Well for Footing Design **if** Seismic Loads Control

# Other SPC B vs. Higher Category Issues

Issues	SPC B	SPC C and D
Seat Width	$N_B = 0.67 \cdot N_{C \& D}$	$N = (12'' + 0.03L + 0.12H) \cdot (0.000125S^2)$
Hinge Zone Confinement	Maximum $s = 6''$	Maximum $s = 4''$
Column Connection Shear Stress	NA	$v \leq 12 \sqrt{f'_c}$ Normal Weight Concrete
Wall Pier Shear Stress	NA	$v \leq 2 \sqrt{f'_c} + \rho_h f_y \leq 8 \sqrt{f'_c}$
Restrainers	NA	Required Between Structure Sections

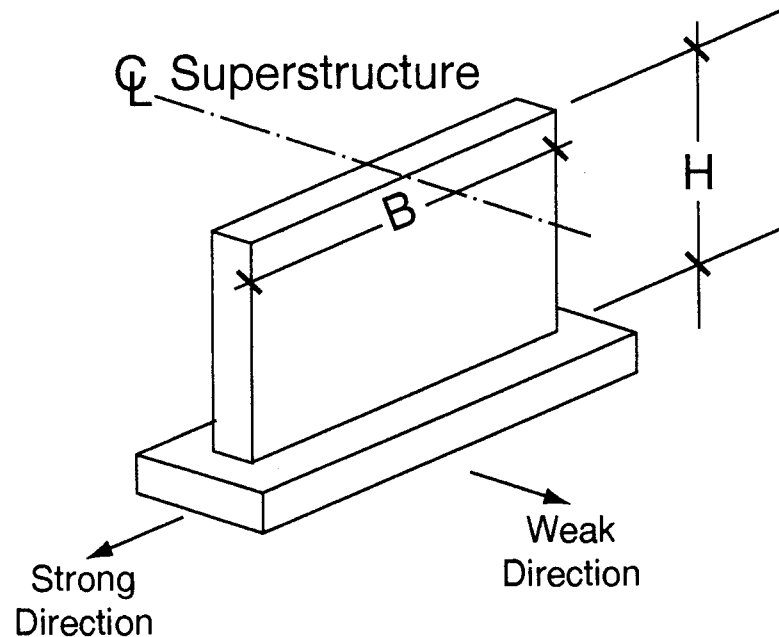
# **Session 7**

## **Column and Pier Design**

---

- **Intended Seismic Behavior**
- **SPC B vs. SPC C and D Requirements**
- **Wall Pier Design**
- **General Detailing Issues**

# Wall Pier Design

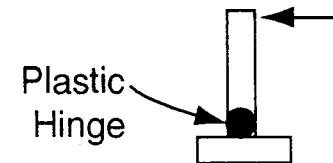
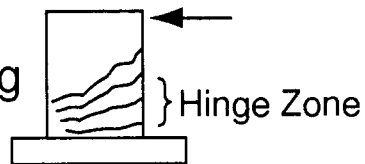


Pier:  $\frac{H}{B} < 2.5$  and Column:  $\frac{H}{B} \geq 2.5$

**Design as Pier (or Wall), or  
Design as a Column**

**Can Develop Plastic Hinging  
at Base of Wall**

**Design as Pier (or Wall)**  
Probably No Plastic Hinging  
at Base of Wall





# Wall Pier Design (continued)

---

- **SPC B**

Strong Direction

$$R = 2$$

Weak Direction

$$R = 2 - \text{Wall}$$

or  $R = 3 - \text{Column}$  { Meet SPC B  
Confinement Requirements

## Wall Pier Design (continued)

---

- **SPC C and D**

Same as B Plus:

- 
- |                  |   |
|------------------|---|
| Column – $R = 3$ | <ul style="list-style-type: none"><li>• Meet SPC C and D Confinement</li><li>• Design for Plastic Hinging</li><li>• <math>\phi=0.5</math></li><li>• Minimum Column Steel,<br/>1% or Arch.</li></ul> |
|------------------|---|
- 

- |                |   |
|----------------|---|
| Wall – $R = 2$ | <ul style="list-style-type: none"><li>• Column Confinement Not Required</li><li>• No Plastic Hinging Design</li><li>• <math>\phi=0.7</math></li><li>• Minimum Horizontal /Vertical<br/>Steel Ratios</li><li>• Limiting Shear Stress</li></ul> |
|----------------|---|

# Minimum Reinforcement for Wall Piers

---

- Is an Issue When Non-Seismic Loads Control
- SPC B, What to Use?

Recommend if

Local Requirements Are Less – Use SPC C and D Values  $\rho_h = \rho_v = 0.0025$

– ACI 318  $\rho_h = 0.0025$   $\rho_v = 0.0015$

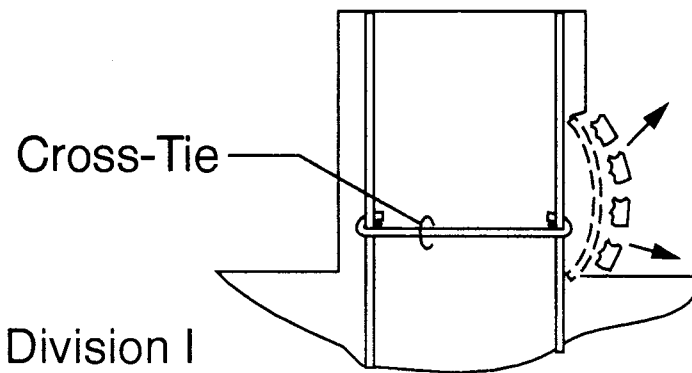
– Local Agency / Durability Issue Mainly

# Cross-Ties in Wall Piers

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- **Design as Column** → Confinement Ratios Control
- **Design as Wall** → No Specific Criteria in Division I-A (SPC B or SPC C and D)

- **Purpose**

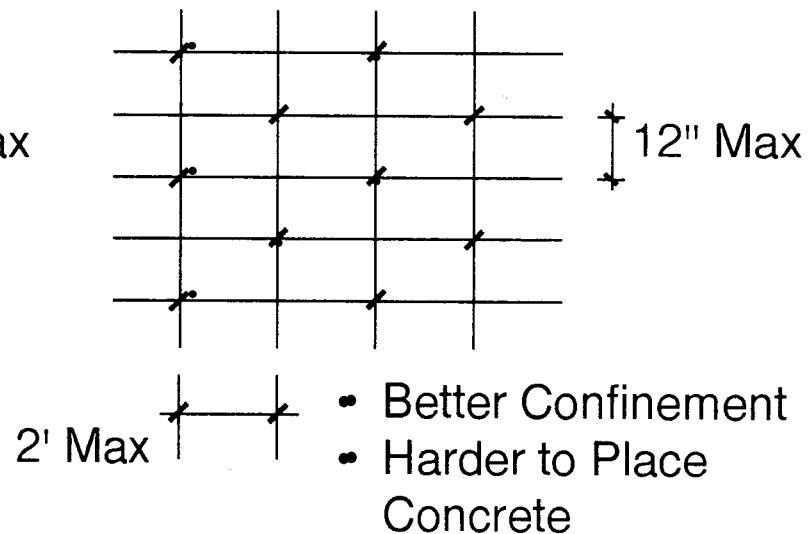
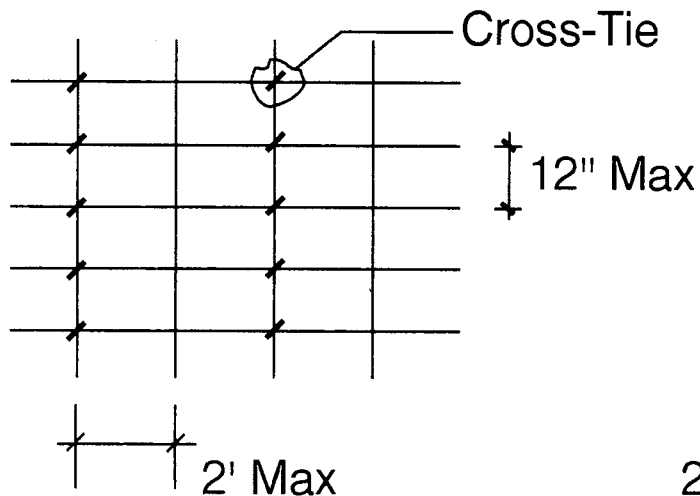


1. Cover Spalls
2. Vertical Bars Tend to Buckle
3. Cross Ties Restrain Vertical Bar

- **Options**
  - Division I
  - Caltrans

# 'Division 1' Cross-Ties

- Spacing of Ties  $s \leq$  Least Member Dimension or 12 Inches
- Longitudinal Bars  $\leq$  2 Feet from a Restrained Bar



**Elevation of Wall Steel**

# **Session 7**

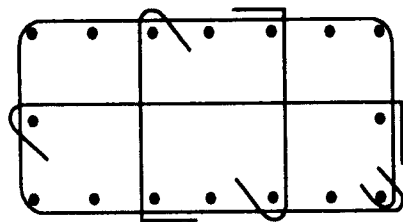
## **Column and Pier Design**

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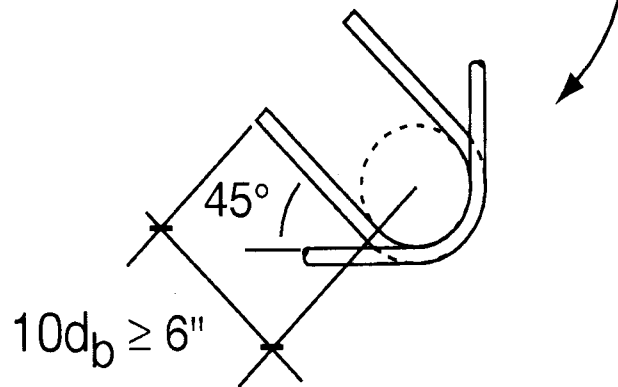
- **Intended Seismic Behavior**
- **SPC B vs. SPC C and D Requirements**
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- **General Detailing Issues**

# Tied-Column Details

Cross Section



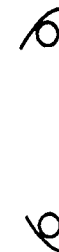
Hook Detail



Anchorage      Confinement



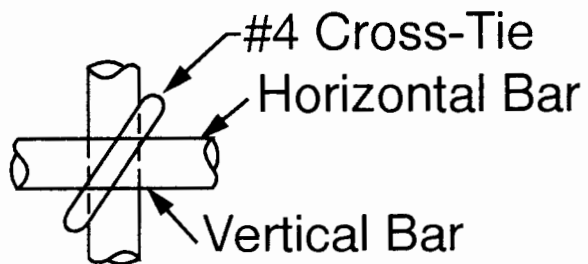
Use Tie



Can't Tie

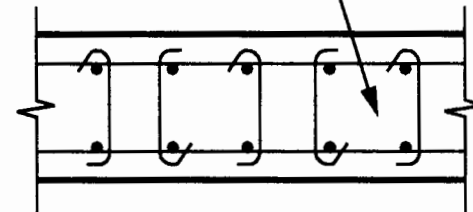
# Cross-Ties in Walls

- **Tie Crosses Both Bars**



- **Alternate Cross-Tie 90° Bends**

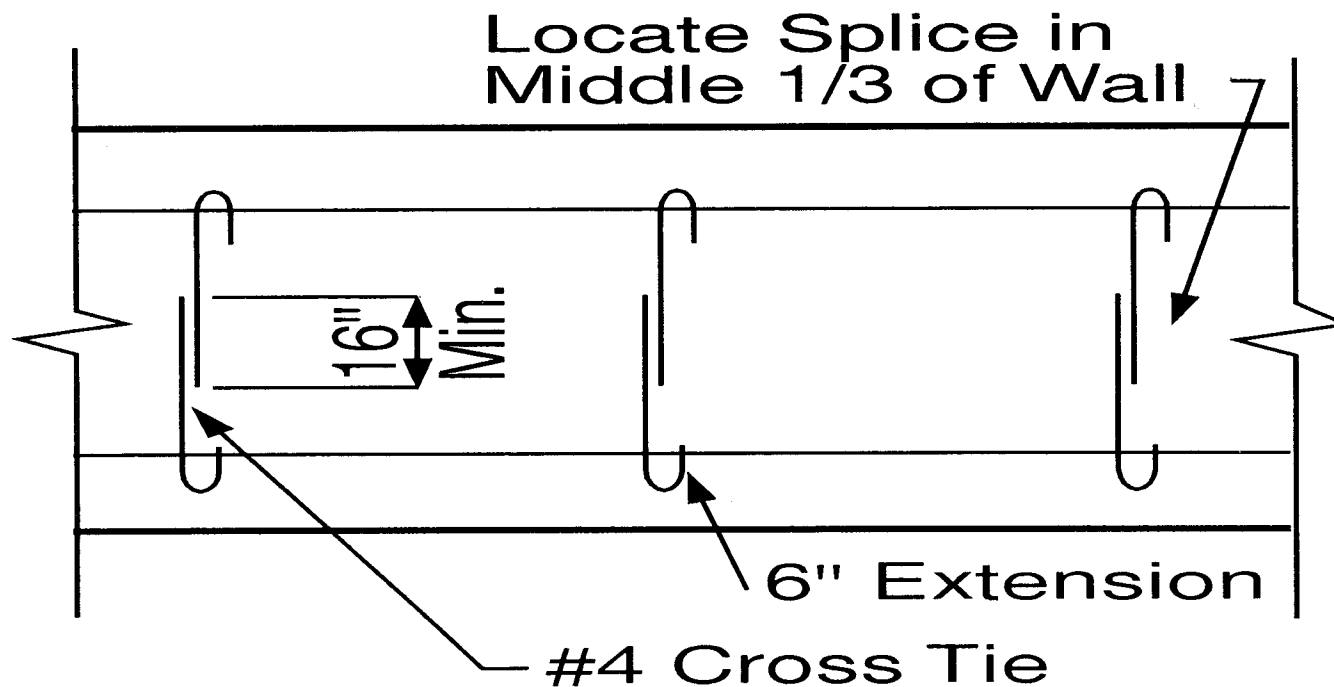
Hooks of Adjacent Cross-Ties Face Each Other to Provide Space for Placing Concrete





## Cross-Ties in Walls (continued)

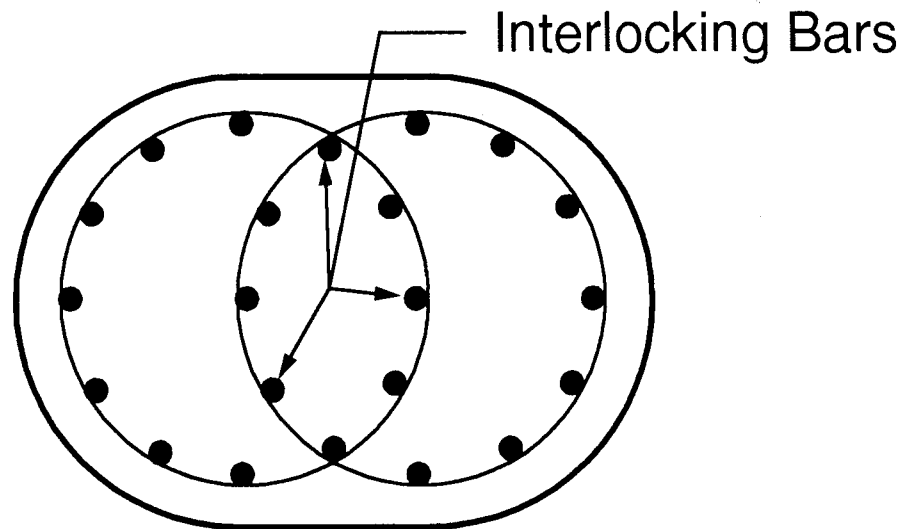
### Alternate to Bending Both Ends



# Interlocking Spiral

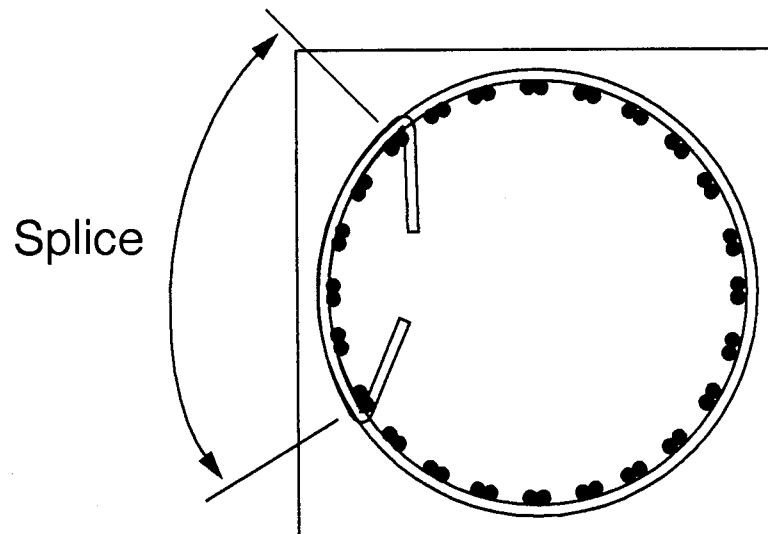
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- Spirals Provide Improved Confinement for Rectangular Columns
- Cross-Ties May Be Required for Shear Strength



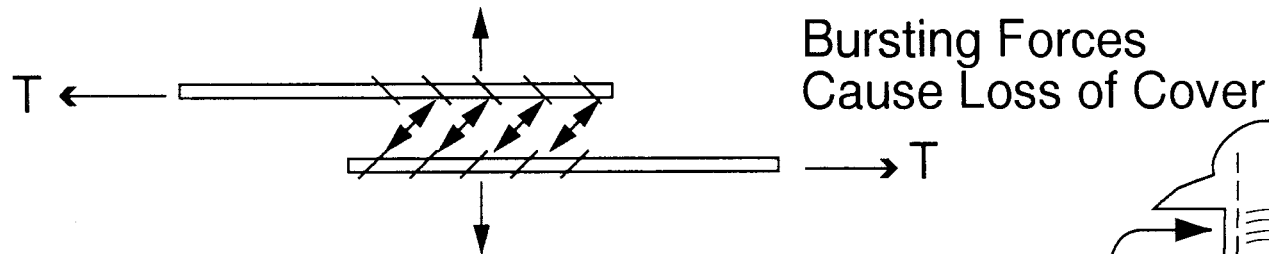
# Spiral Splices

- Hooks Shall Be Placed to Avoid Vertical Reinforcement
- Lap Splices Not Permitted in End Regions
- Alternate: Weld Splice (A706 Steel)

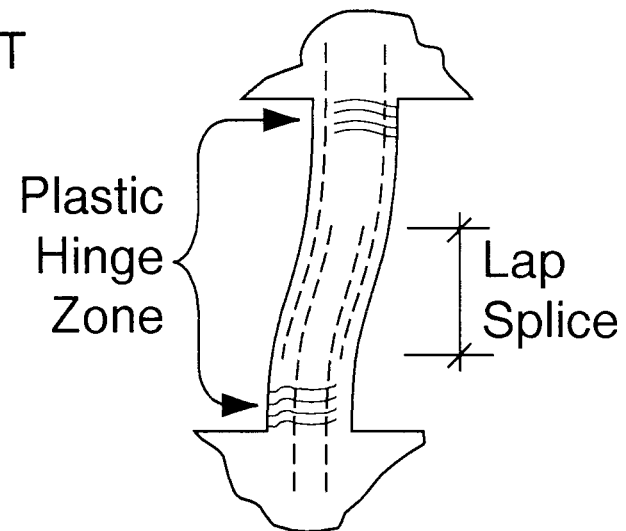


# Reinforcement Splices

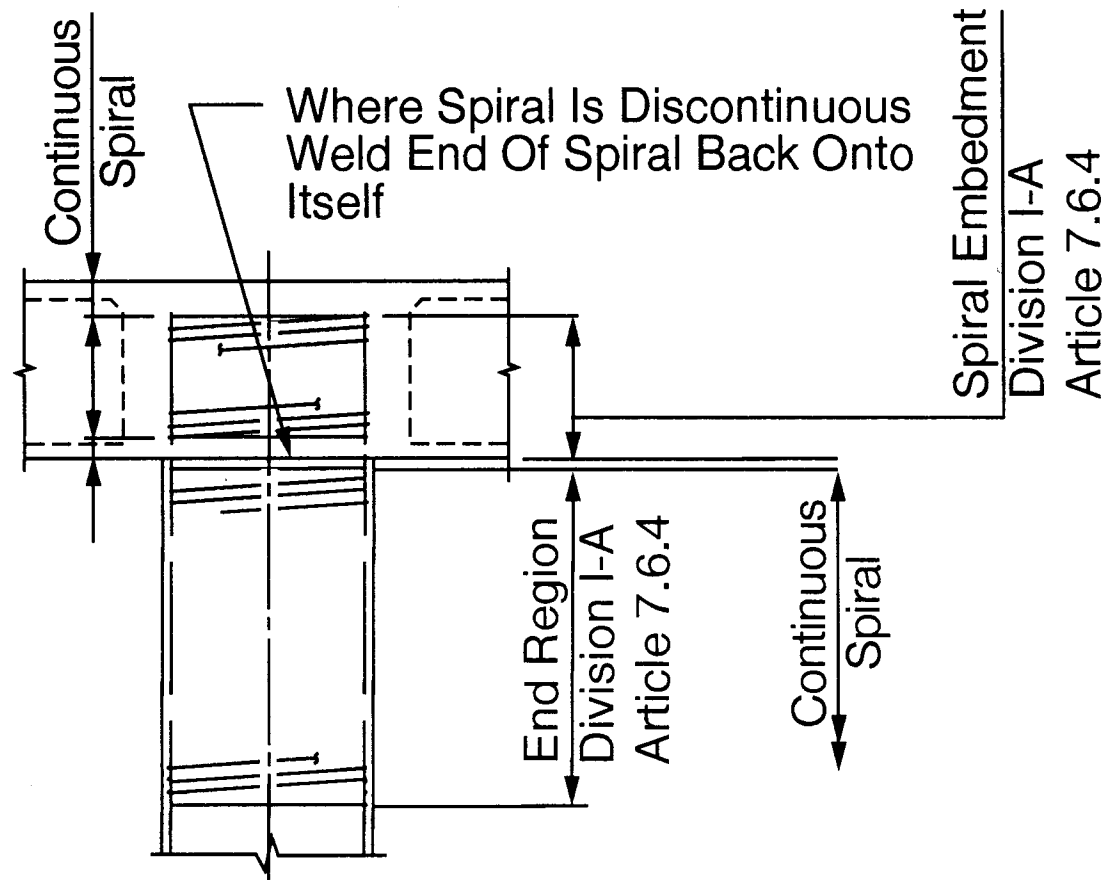
- Don't Splice Longitudinal Steel in Plastic Hinge Zones



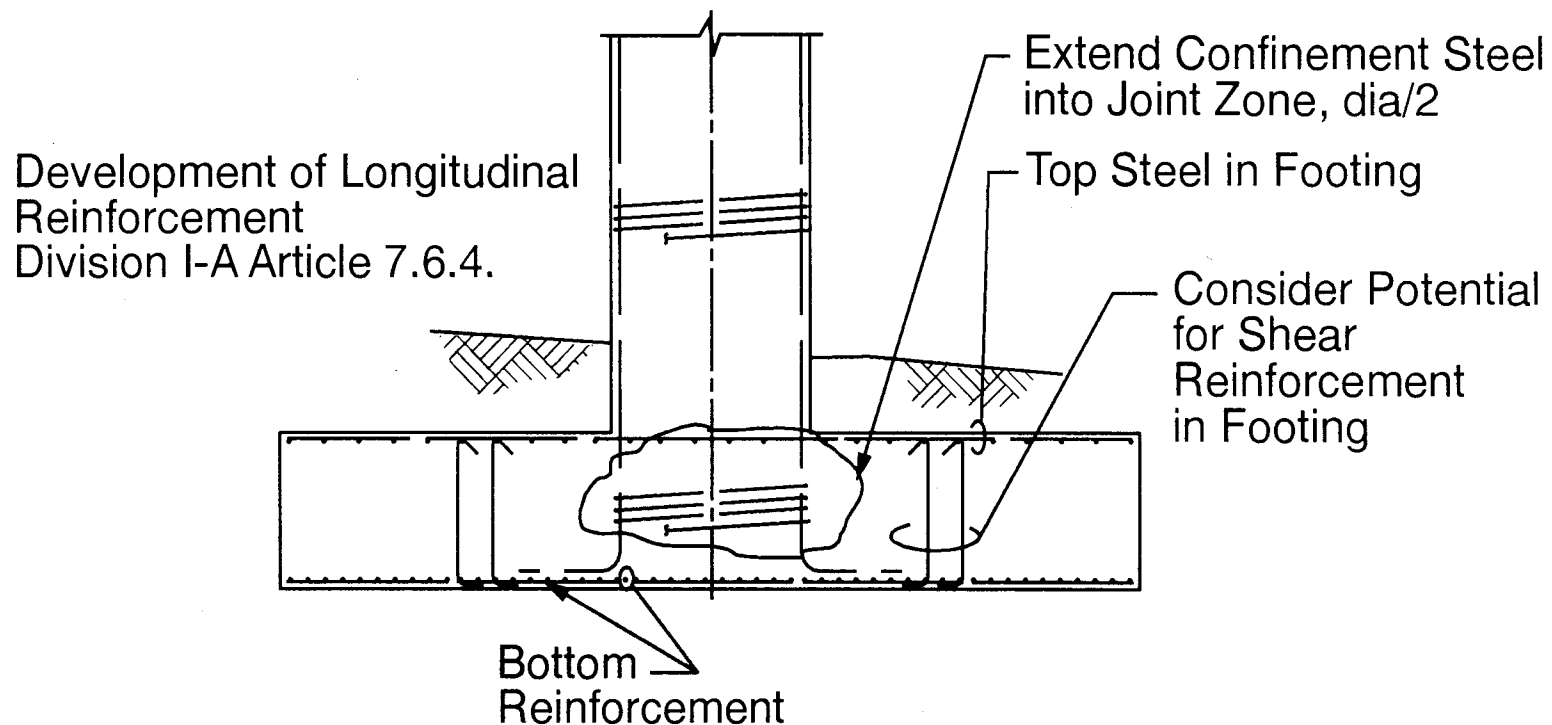
- Lap Splice in Middle Half of Columns (Low Moment Zone)
- Alternate: Welds or Mechanical Connectors (Staggered 24")



# Connection at Cap Beam



# Connection at Footings



# Importance of Details

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UMD-ITV

Seismic Bridge Design Applications

25 April 1996, NHI Course Code No. 13063

# **Session 7**

## **Column and Pier Design**

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- **Questions and Answers**